MATH 245 F20, Exam 2 Questions
(60 minutes, open book, open notes)

1. Freebie.

2. Prove that \( \forall n \in \mathbb{Z}, \) we must have \( \frac{(n+1)(n-2)}{2} \in \mathbb{Z}. \)

3. Let \( x \in \mathbb{R}. \) Prove that TFAE: (a) \( x \) is rational; (b) \( 7x \) is rational; (c) \( x + 1 \) is rational.

4. Prove or disprove: \( \forall x \in \mathbb{R}, \lfloor x \rfloor = -\lceil -x \rceil. \)

5. Prove that \( \forall n \in \mathbb{N}, 9^n > n^3. \)

6. Prove that, for every \( n \in \mathbb{N}, \) the Fibonacci numbers satisfy \( F_{n+3} = 2 + \sum_{i=2}^{n+1} F_i. \)

Pick your favorite, different, real numbers \( b, c \) that are not integers, to use in the rest of the exam.

7. Using your favorite \( b, c: \) solve the recurrence with initial conditions \( a_0 = b, a_1 = c \) and relation \( a_n = 2a_{n-1} - a_{n-2} \) (for \( n \geq 2 \)).

8. Using your favorite \( b, c: \) (i) Prove or disprove that \( n^b = O(n^c) \); and (ii) Prove or disprove that \( n^c = O(n^b) \).