Fall 2019 Math 245 Exam 3

Please read the following directions:

Please write legibly, with plenty of white space. Please fill out the box above as legibly as possible. Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. The use of notes, calculators, or other materials on this exam is strictly prohibited. This exam will begin at 1:00 and will end at 1:50; pace yourself accordingly. Please remain quiet to ensure a good test environment for others. Good luck!
Problem 1. Carefully define the following terms:

a. subset

b. intersection

c. De Morgan’s Law (for sets)

Problem 2. Carefully define the following terms:

a. cardinality

b. set of departure

c. irreflexive

Problem 3. Prove, using definitions, that for all sets $A, B$, we have $(A \cup B) \setminus (A \cap B) \subseteq A \Delta B$. 
Problem 4. Prove or disprove: For all sets $A, B, C$ with $A \subseteq B$, $B \subseteq C$, and $C \subseteq A$, we must have $A = C$.

Problem 5. Prove or disprove: For all sets $A, B$, we must have $2^A \cup 2^B = 2^{A \cup B}$.

Problem 6. Prove or disprove: For all sets $A, B, C$ with $A \subseteq B$ and $B \subseteq C$, we must have $A \times B \subseteq B \times C$.

Problem 7. Prove or disprove: For all sets $A, B$, we must have $A \times B$ equicardinal with $(A \times B) \times A$. 
Problem 8. Let $A, B$ be sets with $A \subseteq B$. Prove or disprove: For all transitive relations $R$ on $B$, we must have $R|_A$ also transitive.

For problems 9,10:
Let $A = \{1, 2, 3, 4\}$ and take $R = \{(1, 1), (1, 2), (2, 3), (3, 4), (4, 1), (4, 3)\}$, a relation on $A$.

Problem 9. Draw the digraph representing $R$. Determine, with justification, whether or not $R$ is each of: reflexive, symmetric, and transitive.

Problem 10. Compute $R \circ R$. Give your answer both as a digraph and as a set.