

**MATH601 Spring 2008**  
**Exam 3: Hyperreal Numbers**

Please read all directions carefully. For this exam, you may not use any books, notes, or other aids apart from a calculator. Please write all solutions clearly and legibly, on separate paper, indicating what work applies to which problem. Cross out incorrect statements, this may improve your grade. Thoroughly justify your solutions. Your grade on each problem will be 25-50 points. If you wish, you may revise one of these problems for extra credit, due on Wednesday 3/12. Your grade on this problem will become the average of your original grade and the revised grade, rounded down. You have 35 minutes; good luck!

1. For  $H$  any positive infinite hyperreal, compute  $st(\sqrt{H^2 + H + 1} - H)$ , or show that it does not exist.
2. Prove the product rule: Given real functions  $f(x), g(x)$ , set  $h(x) = f(x)g(x)$ . Suppose that  $f'(x), g'(x)$  both exist. Then  $h'(x)$  exists, and equals  $f'(x)g(x) + g'(x)f(x)$ . You may use the:

**Increment Theorem** Let  $f(x)$  be any real function.  
If  $f'(x)$  exists, then  $st(f(x)) = st(f(x + \varepsilon))$ , for all infinitesimal  $\varepsilon$ .