

MATH 579 Exam 1; 1/31/12
Please read the exam instructions.

No books or notes are permitted for this exam; calculators are permitted though. Please indicate what work goes with which problem, and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. You have 40 minutes. If you wish, when handing in your exam you may attach your extra credit problem. For more details, see the syllabus.

Choose three problems only from these five.

1. (5-8 points) You throw 20 darts, that all hit a dartboard in the shape of a square of side length 1. You want to draw a bullseye in the shape of a square of side length $1/4$, that contains at least two of your darts. Prove that you can always do this.
2. (5-10 points) Your nemesis chooses a positive integer n . Prove that there are positive integers $a > b$ such that 10 divides $n^a - n^b$.
3. (5-10 points) Your nemesis chooses seven distinct integers. Prove that some pair has either sum or difference (or both) a multiple of 11.
4. (5-10 points) Your nemesis colors each point in the plane red, blue, or green. Prove that there is some rectangle with all four of its corners of the same color.
5. (5-12 points) Your nemesis chooses a sequence of 25 (not necessarily distinct) elements from $\mathbb{Z}_5 \oplus \mathbb{Z}_5$. Prove that some nonempty subsequence sums to $(0, 0)$.

[$\mathbb{Z}_5 \oplus \mathbb{Z}_5$ is a finite abelian group whose elements are ordered pairs (m, n) , where addition is modulo 5 in each coordinate. For example, $(3, 2) + (4, 3) = (2, 0)$.]