MATH 579 Exam 1: 2/3/9

Please read the exam instructions.

Please write your answers on separate paper, indicate clearly what work goes with which problem, and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. You may earn extra credit by submitting by the next class period (Feb. 5), revised solutions to all six problems – for more details, please see the syllabus.

PART I: Choose three problems only from the first five.

1. (5-8 points) Choose 53 distinct integers in $[1, 100]$. Prove that two of them must differ by 13.
2. (5-10 points) Choose 100 points in a unit square. Prove that seven of them must lie within some circle of radius 0.177.
3. (5-10 points) A "lattice point" is a point (x, y) such that both x, y are integers. Prove that among any set of five lattice points there must be two lattice points whose midpoint is also a lattice point.
4. (5-10 points) Prove that some (positive integer) power of 3 ends in 001. (i.e. $3^n = \ldots 001$).
5. (5-12 points) Let $f(x)$ be a polynomial with integer coefficients. Suppose that $f(x)$ has nine different integer roots. Prove that, for every integer x, if $f(x) \in [-11, 11]$, then $f(x) = 0$.

PART II:

6. (10-20 points) Let S be a set with n elements. Choose over half of the subsets of S; prove that two of the subsets you've chosen have one a subset of the other.