Math 579 Final Exam: 5/10/7

Please read the exam instructions.

Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Each problem is worth a minimum of 8 points, and a maximum that is indicated. You have 55 minutes. *Choose four problems.*

1. (12 points) Recall that the Fibonacci numbers are $F_1 = F_2 = 1, F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. Prove that they satisfy $F_n \geq (1.4)^n$, for all integer $n \geq 6$.

2. (12 points) First, find the number of integer partitions of 100 into five distinct even parts. Second, find the number of integer partitions of 100 into four distinct odd parts.

3. (15 points) Find two constants b, c such that the sequence $a_n = 10^n - 2^n$ satisfies the recurrence relation $a_n = ba_{n-1} + ca_{n-2}$.

4. (15 points) Use the Inclusion-Exclusion principle to find the number of primes in $[120] = \{1, 2, \ldots, 120\}$.

5. (15 points) There are 123 people at a party, who together ate 3700 appetizers. Prove that there must be three people who ate the same number of appetizers.

6. (15 points) Find a closed-form formula for the number of ordered pairs (A, B) where A, B are subsets of $[n]$ and $A \cap B = \emptyset$.
   (note: A, B, or both are permitted to be empty)

7. (18 points) Let D_n represent the number of permutations on $[n]$ with no fixed points, and C_n represent the number of permutations on $[n]$ with exactly one fixed point. For which n is $D_n > C_n$?