MATH 579: Combinatorics
Unit 6 Definitions

graph A graph $G = (V, E)$ consists of a set of vertices* V and a set of edges E. Each edge is a set consisting of a pair of vertices. Note: for us, an edge must contain two distinct vertices, and all edges must be different. This is often called a "simple graph" in the literature.

incident If edge e contains vertex v, then we say each is incident with the other.

adjacent If edge e contains vertices u, v, then we say that vertices u, v are adjacent.

degree The degree of a vertex is the number of edges that is incident with it.

walk A walk is a list $v_0, e_1, v_1, e_2, \ldots, e_k, v_k$ where for $1 \le i \le k$, $e_i = \{v_{i-1}, v_i\}$. Its length is k.

closed A walk is closed if $v_0 = v_k$.

trail A trail is a walk with no edge repeated.

path A path is a walk with no edge or vertex repeated.

cycle A cycle is a closed path.

even graph A graph is even if all of its vertices are of even degree.

Eulerian A closed trail is Eulerian if it contains every edge of the graph. A graph is Eulerian if it has an Eulerian trail.

Hamiltonian A path or cycle is Hamiltonian if it contains every vertex of the graph.

subgraph $G' = (V', E')$ is a subgraph of $G = (V, E)$ if $V' \subseteq V$ and $E' \subseteq E$.

connected A graph is connected if there is a path between any pair of vertices.

component A component of a graph is a maximal connected subgraph. A component is nontrivial if it has at least one edge.

K$_n$ The complete graph K_n consists of n vertices and every possible edge between them.

clique, coclique A clique is a complete subgraph. A coclique is a set of vertices containing no edges between them.

bipartite Graph $G = (V, E)$ is bipartite if there is a partition $V = V_1 \cup V_2$ and every edge contains exactly vertex from V_1 and one from V_2.

K$_{m,n}$ The complete bipartite graph $K_{m,n}$ has partition $V = V_1 \cup V_2$ with $|V_1| = m$, $|V_2| = n$, and every possible edge between V_1 and V_2.

C$_n$ The cycle graph C_n contains n vertices, edges to form a cycle of length n, and nothing else.

Petersen graph The vertices are the two-element subsets of $\{a, b, c, d, e\}$. An edge contains $\{u, v\}$ with $\{x, y\}$ if these two sets are disjoint.

decomposition A decomposition of a graph is a partition of the edges (each part forms a subgraph).

graph isomorphism Given graphs $G = (V, E)$ and $G' = (V', E')$, an isomorphism from G to G' is a bijection $f : V \to V'$ satisfying the property $\{u, v\} \in E \leftrightarrow \{f(u), f(v)\} \in E'$.

tree A tree is a connected graph containing no cycles.

pendant A vertex is pendant (also called a leaf) if it has degree 1.

spanning tree A spanning tree is a subgraph, on all the vertices, that is a also a tree.

*Singular of "vertices" is vertex. ~~vertice~~