

MATH 579: Combinatorics
Homework 1: Due Sep. 6

For these next problems, a “word” is a string of letters, drawn from the 26 options a, \dots, z . n represents an arbitrary natural number; solve the problems for all n . You might want to check your answer for a few small values of n .

1. How many words are there of length n ?
2. How many words are there of length n , with all different letters?
3. How many words are there of length n , using each of the 26 letters at least once?
4. How many words are there of length n , with no vowels?
5. How many words are there of length n , with at least one vowel and at least one consonant?
6. How many words are there of length n , with the first three letters vowels, and the remaining letters consonants?
7. How many words are there of length n , with exactly three a 's?
8. How many words are there of length n , with exactly three a 's, appearing consecutively?
9. How many words are there of length n , with no two consecutive letters being the same?
10. How many words are there of length n , whose first and last letters are the same, and also second and second-to-last letters are the same, and so on?

For these next problems, we are shopping. The store has 26 items for sale, numbered 1 to 26. We can buy an item more than once, and the order in which we buy items does not matter.

11. How many ways are there of buying n items?
12. How many ways are there of buying n items, all numbered with primes?
13. How many ways are there of buying n items, all different?
14. How many ways are there of buying n items, ensuring that we buy all 26 items at least once?
15. How many ways are there of buying n items, all different, ensuring that we do not buy two items with consecutive numbers. (Hint: $a_1 < a_2 - 1 < a_3 - 2 < \dots$)