MATH 579 Exam 2; 9/17/13

Please read the exam instructions.

No books or notes are permitted for this exam; calculators are permitted though. Please indicate what work goes with which problem, and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. You have 40 minutes. If you wish, when handing in your exam you may attach your extra credit problem. For more details, see the syllabus.

Choose three problems only from these five.

1. (5-8 points) Prove that $\sum_{i=1}^{n} i(i-3) = \frac{(n-4)n(n+1)}{3}$.
2. (5-10 points) Let $a_1 = 1, a_2 = 5$, and $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 2$. Prove that $a_n = 2^n + (-1)^n$.
3. (5-10 points) Recall that F_i denotes the Fibonacci numbers, i.e. $F_1 = F_2 = 1$ and $F_j + F_{j+1} = F_{j+2}$ for $j \geq 1$. Prove that $\sum_{i=1}^{n} F_i^2 = F_n F_{n+1}$, for all $n \in \mathbb{N}$.
4. (5-10 points) Use induction to prove that $\frac{d}{dx} x^n = nx^{n-1}$, for all $n \in \mathbb{N}$.
5. (5-12 points) A *tree* is a connected simple[1] finite graph with no cycles. Prove that every tree on n vertices must have exactly $n-1$ edges. You may use freely the following result: **Theorem:** Every tree with at least two vertices has at least two leaves.

[1]Simple means: no loops (edges from a vertex to itself), and no multiple edges (at most one edge between two given vertices).