Math 524 Exam 4: 10/7/8

Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Each problem is worth 10 points. You have approximately 30 minutes.

For each of the following vector spaces V and linear operators L:

1. Find all eigenvalues.
2. Find a basis for each eigenspace.
3. Determine all algebraic and geometric multiplicities.
4. Is the operator diagonalizable?

A. $V = \mathbb{R}^3$, $L(x) = \begin{bmatrix} 3 & 0 & 0 \\ 1 & 1 & -1 \\ -2 & 4 & 5 \end{bmatrix} x$

B. $V = M_{2,2}(\mathbb{R})$, the set of all 2×2 real matrices. We have $V = W_1 + W_2$, an internal direct sum, where $W_1 = \{A : A = A^T\}$ is the subspace of symmetric matrices, and $W_2 = \{A : A = -A^T\}$ is the subspace of skew-symmetric matrices. L is the operator that projects from V to W_1.

C. $V = C^1(\mathbb{R})$, the set of continuously differentiable functions on the real line, $L(f) = t\frac{df}{dt}$