Math 524 Exam 3: 9/25/8

Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Each problem is worth 10 points. You have approximately 30 minutes.

Problems 1-4 are for the vector space $\mathbb{R}_2[t]$, real polynomials of degree at most 2. We define $L : \mathbb{R}_2[t] \to \mathbb{R}_2[t]$ via $L(f) = (t-1)\frac{df}{dt}$.

1. Directly calculate $[L]_E$, for the basis $E = \{1, t, t^2\}$.
2. Directly calculate $[L]_B$, for the basis $B = \{1, t-1, (t-1)^2\}$.
3. Calculate P_{BE}, P_{EB}, and demonstrate the relationship between them and $[L]_B, [L]_E$.
4. Find a basis for the kernel of L. Find a basis for the range of L.
5. Consider the operator $M : \mathbb{R}[t] \to \mathbb{R}[t]$, an operator on real polynomials given by $M(f) = \frac{d^2 f}{dt^2}$. Calculate the nullity of M, and prove that M is onto. Why doesn't this contradict the Dimension Theorem?