

## Math 524 Exam 2: 9/16/8

Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Each problem is worth 10 points. You have approximately 30 minutes.

All problems are for the vector space  $\mathbb{R}_2[t]$ , real polynomials of degree at most 2. We define  $V = \{p(t) : p(1) = 0\}$ , a subspace of  $\mathbb{R}_2[t]$ .

1. Let  $A = \{a_1, a_2\}$  for  $a_1 = t - 1, a_2 = t^2 - 1$ .  
Let  $B = \{b_1, b_2\}$  for  $b_1 = t^2 + t - 2, b_2 = t^2 + 2t - 3$ .  
Prove that  $A$  and  $B$  are each bases of  $V$ .
2. Calculate  $[3t^2 - 5t + 2]_A$ .
3. Calculate  $P_{BA}$ .
4. Use the results of the previous two problems to calculate  $[3t^2 - 5t + 2]_B$ .
5. Let  $W = \{at : a \in \mathbb{R}\}$ . This is a subspace of  $\mathbb{R}_2[t]$ . Prove that  $\mathbb{R}_2[t]$  is the internal direct sum of  $V$  and  $W$ .