An isometry (also called a symmetry) of an object $A$ in $\mathbb{R}^n$ is a function $f : \mathbb{R}^n \to \mathbb{R}^n$ satisfying two properties:

1. For all $v \in A$, $f(v) \in A$, and
2. For all $v, u \in \mathbb{R}^n$, $|f(v) - f(u)| = |v - u|$.

We say that $f$ preserves $A$ (property 1), and $f$ preserves distances (property 2). Note: $v, u$ above are vectors.

A key property of isometries is that they preserve structure. An isometry maps a corner of $A$ to a corner of $A$, of the same degree. It maps an edge of $A$ to an edge of $A$, of the same length, and the same degrees at the ends. It maps a face of $A$ to a face of $A$, with the same number of sides and the same area.

1. Explicitly write down four different isometries of $A = \{(0, 0)\}$ in $\mathbb{R}^2$.
   Example: $f(x, y) = (y, x)$.

2. Explicitly write down some $f : \mathbb{R}^2 \to \mathbb{R}^2$ that preserves $A = \{(0, 0)\}$ but does NOT preserve distances.

3. Explicitly write down some $f : \mathbb{R}^2 \to \mathbb{R}^2$ that preserves distances but does NOT preserve $A = \{(0, 0)\}$.

4. Explicitly write down all the isometries in $\mathbb{R}^2$ for $A$ the line segment between $(-1, 0)$ and $(1, 0)$. Justify why your list is complete.

5. Explicitly write down all the isometries in $\mathbb{R}^2$ for $A$ the line segment between $(-1, 0)$ and $(2, 0)$. Justify why your list is complete.

6. Explicitly write down all the isometries in $\mathbb{R}^2$ for $A$ the triangle with vertices $(-1, 0)$, $(1, 0)$, $(0, 1)$. Justify why your list is complete.

7. How many isometries are there in $\mathbb{R}^2$ for $A$ the triangle with vertices $(-1, 0)$, $(1, 0)$, $(0, \sqrt{3})$. Justify why your list is complete. Why is the answer different from the previous problem?

8. Explicitly write down all the isometries in $\mathbb{R}^2$ for $A$ the unit circle $x^2 + y^2 = 1$. Justify why your list is complete. Hint: polar coordinates.

9. Explicitly write down all the isometries in $\mathbb{R}^3$ for $A$ the line segment between $(0, 0, -1)$ and $(0, 0, 1)$. Justify why your list is complete. Hint: cylindrical coordinates.