MATH 521A: Abstract Algebra
Homework 6: Due Oct. 25

1. Let R, S, T be rings, with S, T both subrings of R. Suppose that S has the special property that for every $s \in S$ and every $r \in R$, we have both $sr \in S$ and $rs \in S$. Set $S+T = \{s+t : s \in S, t \in T\}$, a subset of R. Prove that $S+T$ is a subring of R. [This is really a Chapter 3 question.]

2. Consider the polynomial ring $\mathbb{Z}_9[x]$, and the nine elements $\{3x+0, 3x+1, \ldots, 3x+8\}$. Determine which are units and which are zero divisors.

3. Consider the polynomial ring $\mathbb{Z}_9[x]$, and the nine elements $\{0x+3, 1x+3, \ldots, 8x+3\}$. Determine which are units and which are zero divisors.

4. Let R be a ring, and $k \in \mathbb{N}$. Define $x^kR[x] = \{x^kf(x) : f(x) \in R[x]\}$. Prove that $x^kR[x]$ is a subring of $R[x]$.

5. Let F be a field. Determine explicitly which elements of $F[x]$ are in the subring $x^3F[x] + x^5F[x]$. (refer to exercises 1,4)

6. Working in $\mathbb{Q}[x]$, find $\gcd(a(x), b(x))$, for $a(x) = x^3+x^2+x+1$, $b(x) = x^4-2x^2-3x-2$.

7. Working in $\mathbb{Z}_2[x]$, find $\gcd(a(x), b(x))$, for $a(x) = x^3+x^2+x+1$, $b(x) = x^4-2x^2-3x-2$.

8. Working in $\mathbb{Z}_5[x]$, find $\gcd(a(x), b(x))$, for $a(x) = x^3+x^2+x+1$, $b(x) = x^4-2x^2-3x-2$.

9. Working in $\mathbb{Q}[x]$, let $a(x) = x^2-5x+6$, $b(x) = x^3-x^2-2x$. Find $u(x), v(x)$ such that $\gcd(a(x), b(x)) = a(x)u(x) + b(x)v(x)$.

10. Working in $\mathbb{Z}_3[x]$, let $a(x) = x^2-5x+6$, $b(x) = x^3-x^2-2x$. Find $u(x), v(x)$ such that $\gcd(a(x), b(x)) = a(x)u(x) + b(x)v(x)$.