MATH 521A: Abstract Algebra

Homework 4: Due Oct.4

1. Use the generalized Euclidean algorithm (with $101, 999$) to find the congruence class satisfying the linear modular equation $101x \equiv 1 \pmod{999}$.
2. Find all congruence classes satisfying the linear modular equation $24x \equiv 10 \pmod{35}$.
3. Find all congruence classes satisfying the linear modular equation $25x \equiv 10 \pmod{35}$.
4. Find all congruence classes satisfying the linear modular equation $25x \equiv 11 \pmod{35}$.
5. Let R be a commutative ring with identity. Prove that no element can be both a unit and a zero divisor.
6. Let R be a commutative ring with identity. Let $a_1, a_2 \in R$ be units, and $b_1, b_2 \in R$ be nonzero nonunits. Prove that a_1a_2 is a unit, while a_1b_1 and b_1b_2 are nonunits.
7. Let R be a commutative ring with identity. Let $a_1, a_2 \in R$ be zero divisors, and $b_1, b_2 \in R$ be nonzero and not zero divisors. Prove that a_1b_1 is a zero divisor, while b_1b_2 is not a zero divisor. Must a_1a_2 be a zero divisor?
8. Let R be a ring, with S a subring. Prove that $0_R = 0_S$, and that every zero divisor of S is also a zero divisor of R.
9. Let R be a ring, with S a subring. Suppose that they share a multiplicative neutral element, i.e. $1_R = 1_S$. Suppose that $a \in S$, and that a is a unit in S. Prove that a is a unit in R.
10. Give an example of a commutative ring with identity R, with subring S, where the rings do NOT share a multiplicative neutral element. That is, with $1_R \neq 1_S$. Further, find an element $a \in S$ that is a unit in S but NOT a unit in R.