# Math 254 Fall 2014 Exam 2a Solutions

1. Carefully state the definition of matrix space $M_{m,n}$. Give a set of two vectors, drawn from $M_{2,2}$.

   $M_{m,n}$ is the set of all matrices with real coefficients, $m$ rows, and $n$ columns. A correct example must be be a set (with curly braces) containing two different $2 \times 2$ matrices. One possible answer is $\{\left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right), \left(\begin{smallmatrix} 1 & 2 \\ 3 & 4 \end{smallmatrix}\right)\}$

2. List, in any order, the three elementary operations that leave unchanged the solution set to a system of linear equations.

   E1: Interchange two equations. E2: Multiply any equation by a *nonzero* constant. E3: Add a multiple of one equation to another.

3. Solve the following system of equations using back-substitution. Show your work.

$$\begin{aligned} 6x_1 + 3x_2 + 2x_3 - x_4 &= 4 \\ 5x_2 + 3x_3 + 2x_4 &= 5 \\ -7x_3 + 3x_4 &= 15 \\ 2x_4 &= 10 \end{aligned}$$

   Step 1: $2x_4 = 10$ gives $x_4 = 5$. Step 2: $-7x_3 + 3(5) = 15$ gives $x_3 = 0$. Step 3: $5x_2 + 3(0) + 2(5) = 5$ gives $x_2 = -1$. Step 4: $6x_1 + 3(-1) + 2(0) - (5) = 4$ gives $x_1 = 2$. In summary, $(x_1, x_2, x_3, x_4) = (2, -1, 0, 5)$, a unique solution.

4. Find the line of best fit for the following set of points: $\{(-2, 2), (1, 1), (3, 3), (4, 4)\}$.

   We calculate $N = 4, \sum x = 6, \sum x^2 = 4 + 1 + 9 + 16 = 30, \sum y = 10, \sum xy = -4 + 1 + 9 + 16 = 22$. This gives the system $\{4b + 6m = 10, 6b + 30m = 22\}$, which has unique solution $b = 2, m = \frac{1}{3}$. Hence the desired line is $y = \frac{1}{3}x + 2$.

5. Give a system of three equations in unknowns $x, y$ with no solutions, with the additional property that none of the three lines has the same slope as either of the others.

   Many solutions are possible; the key is to take three lines forming a triangle in the plane. $\{x = 2, y = 3, x+y = 1\}$ is a simple example. Another is $\{x+y = 1, x-y = 0, x = 10\}$.