

Name: \_\_\_\_\_

## Math 254 Fall 2014 Exam 12

Please read the following directions:

Please print your name in the space provided, using large letters, as “First LAST”. Books, notes, calculators, and other aids are not permitted on this exam. Please write legibly, with plenty of white space. Please put your answers in the designated areas. Show all necessary work in your solutions; if you are unsure, show it. Cross out work you do not wish graded; incorrect work can lower your grade. All problems are worth 5-10 points; your total will be scaled to the standard 100 point scale. You have approximately 30 minutes.

Extra credit may be earned by handing in revised work in class on Wednesday 12/10; for details see the syllabus. You will find this exam on the instructor’s webpage later today.

1. Carefully state the definition of “basis”. Give a basis for  $M_{2,2}$ .

2. Give an example of a quadratic form  $q(x, y)$  such that there are two vectors  $u = (x_u, y_u)$ ,  $v = (x_v, y_v)$  with  $q(u) = 0$  and  $q(v) = 0$  but  $q(u + v) \neq 0$ .

The remaining problems all concern  $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 6 \\ 3 & 6 & 9 \end{pmatrix}$ .

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3. Find invertible matrix  $P$  and diagonal matrix  $D$  such that  $D = P^T A P$ .

4. Use  $D$  to find the rank and signature of  $A$ . Is  $A$  positive definite?

5. Consider the quadratic form  $q(x, y, z) = x^2 + y^2 + 9z^2 + 4xy + 6xz + 12yz = 1$ , and the surface given by  $q(x, y, z) = 1$ . Use  $A, P, D$  to diagonalize this quadratic form. Write the surface in the new variables, and show the relationship between the old and new variables. What is the name of this surface?