Math 254 Fall 2013 Exam 9 Solutions

1. Carefully state the definition of "vector space". Give two three-dimensional examples.

   A vector space is a collection of objects (called vectors), a set of scalars (typically $\mathbb{R}$), and a way to add vectors and multiply vectors by scalars. Two familiar three-dimensional examples are $\mathbb{R}^3$ and $P_2(t)$.

2. Consider the linear mapping $g : \mathbb{R}^3 \to P_2(t)$ given by $g((a,b,c)) = a + (b+c)t + at^2$. Find a basis for the kernel of g, and find a basis for the image of g.

   If (a,b,c) is in the kernel of g, then $g((a,b,c)) = a + (b+c)t + at^2 = 0$, so $a = 0, b+c = 0, a = 0$. This is a one-dimensional space, with basis $\{(0,1,-1)\}$.

   By the rank-nullity theorem, $dim(Im\ g) + dim(Ker\ g) = dim(\mathbb{R}^3)$, so $dim(Im\ g) = 2$ and any basis for $Im\ g$ will consist of two (linearly independent) vectors. One example is $\{1+t^2, t\}$.

Fill in each of the following blanks with the best choice from:
(A) Scalar, (B) Vector, (C) Finite Set of Vectors, (D) Vector Space, (E) None of the above.

3. $P_2(t)$ is a D.
4. $\mathbb{R}^3$ is a D.
5. $M_{2,2}$ is a D.
6. $(1,2,3)$ is a B in $\mathbb{R}^3$.
7. $\{(1,2,3)\}$ is a C in $\mathbb{R}^3$.
8. $\{1,2,3\}$ is a E in $\mathbb{R}^3$.
9. $1+2t$ is a E in $P_2(x)$.
10. $1+2t$ is a B in $P_2(t)$.
11. $\{1+2t\}$ is a C in $P_2(t)$.
12. $(1,2t)$ is a E in $P_2(t)$.
13. $\{(1,2t)\}$ is a E in $P_2(t)$.
14. $\{1,2t\}$ is a C in $P_2(t)$.
15. We take the span of a C.
16. A basis of $P_2(t)$ is a C.
17. In $M_{2,3}$, a C can be dependent.
18. In $M_{2,3}$, a D can be a subspace.
19. A norm takes as input a B.
20. A norm produces as output a A.
21. An inner product inputs two B.
22. An inner product outputs a A.
23. The domain of a linear transformation is a D.
24. The input to a linear transformation is a B.
25. The sum of two vectors is a B.
26. The sum of two scalars is a A.
27. The sum of a vector and a scalar is a E.
28. The sum of a vector and a vector space is a E.
29. The sum of two subspaces is a D.
30. In $P_2(t)$, the product of a scalar and a vector is a B.
31. In $P_2(t)$, the product of two scalars is a A.
32. In $P_2(t)$, the product of two vectors is a E.
33. The intersection of two subspaces is a D.
34. The intersection of two sets of vectors is a C.
35. The intersection of two vectors is a E.
36. The rowspace of a matrix is a D.
37. The kernel of a linear transformation is a D.
38. The image of a linear transformation is a D.
39. The solution set to a homogeneous linear system is a D.
40. The span of a set of vectors is a D.