

Name:

Math 254 Fall 2012 Exam 8

Please read the following directions:

Please print your name in the space provided, using large letters, as “First LAST”. Books, notes, calculators, and other aids are not permitted on this exam. Please write legibly, with plenty of white space. Please put your answers in the designated areas. Show all necessary work in your solutions; if you are unsure, show it. Cross out work you do not wish graded; incorrect work can lower your grade. All problems are worth 5-10 points; your total will be scaled to the standard 100 point scale. You have approximately 30 minutes.

Extra credit may be earned by handing in revised work in class on Friday 11/9; for details see the syllabus. You will find this exam on the instructor’s webpage soon.

1. Carefully state the definition of “linear transformation”. Give two examples, each on \mathbb{R}^3 .

2. Consider the linear map $F : M_{2,2} \rightarrow M_{2,2}$ given by $F(A) = A - A^T$. Find a basis for its kernel, and a basis for its image.

3. What are the possible ranks and nullities of linear transformations $F : \mathbb{R}^2 \rightarrow \mathbb{R}^3$? Give an example of each possible combination, and indicate which are one-to-one and which are onto.

4. Consider the linear transformation $F : \mathbb{R}^2 \rightarrow P_2(x)$ given by $F((a, b)) = \int_0^x (at + b) dt$. Find its rank and nullity, and a basis for its image.

5. Suppose that $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation, and $F \circ F \circ F \circ F = I_2$ (identity). Prove that the nullity of F is 0, and find such an F .