

Name: _____

Math 254 Fall 2012 Exam 7

Please read the following directions:

Please print your name in the space provided, using large letters, as “First LAST”. Books, notes, calculators, and other aids are not permitted on this exam. Please write legibly, with plenty of white space. Please put your answers in the designated areas. Show all necessary work in your solutions; if you are unsure, show it. Cross out work you do not wish graded; incorrect work can lower your grade. All problems are worth 5-10 points; your total will be scaled to the standard 100 point scale. You have approximately 30 minutes.

Extra credit may be earned by handing in revised work in class on Friday 11/2; for details see the syllabus. You will find this exam on the instructor’s webpage soon.

1. Carefully state the definition of “vector space”. You need not write out all the properties in detail. Give two examples, each six dimensional.

2. Let $u = (2, -1, 0)$, a vector in \mathbb{R}^3 . Compute $\|u\|_1, \|u\|_2, \|u\|_3, \|u\|_\infty$.

3. Consider the vector space $M_{2,2}$ with the usual inner product $\langle X, Y \rangle = \text{tr}(X^T Y)$. Set $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, and $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. Find B_1, B_2 such that $B = B_1 + B_2$, B_1 is a multiple of A , and B_2 is orthogonal to A .

The remaining two questions concern vector space $V = P_1(x)[0, 1]$, the set of polynomials of degree at most 1 on interval $[0, 1]$ with inner product $\langle u(x), v(x) \rangle = \int_0^1 u(x)v(x)dx$. Let $S = \{s_1, s_2\}$ for $s_1(x) = \sqrt{3}x$, $s_2(x) = -3x + 2$.

4. Prove that S is an orthonormal set (hence a basis).

5. Find the Fourier coefficients of $u(x) = x + 1$ with respect to S .