

### Math 254-1 Exam 8: 11/10/8

Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on **the attached page only** (front and back if necessary). Indicate clearly what work goes with which problem. Cross out work you do not wish graded; incorrect work can lower your grade. You may use this first page as scratch paper; keep it for your records. Show all necessary work in your solutions; if you are unsure, show it. Extra credit may be earned by handing in revised work in class on Wednesday 11/12; for details see the syllabus. Each problem is worth 10 points; your total will be doubled to fit the standard 100 point scale. You have approximately 30 minutes.

1. Carefully define the linear algebra term “linear mapping”. Give two examples on  $\mathbb{R}^2$ .
2. Give any inner product on  $\mathbb{R}^2$ , OTHER than the dot product. Use your inner product to calculate  $\langle u, v \rangle$  for  $u = (1, 1)^T, v = (2, 3)^T$ .
3. Find two different functions  $f, g$  on  $\mathbb{R}$ , with  $f \circ f = g \circ g = 1_{\mathbb{R}}$ .
4. Consider all possible linear mappings from  $\mathbb{R}^4$  to  $\mathbb{R}^2$ . What are the possible nullities and ranks of these? Give an example function for each possible combination, and indicate which functions are one-to-one and which are onto.
5. Consider the mapping  $F : \mathbb{R}_2[t] \rightarrow \mathbb{R}^2$  given by  $F(p(t)) = (p(2), p(-1))$ . Calculate  $F(p(t))$  for  $p(t) = t^2 + 3t - 1$ . Determine whether  $F$  is linear.

**Please hand in ONLY the second page; keep this first page.**

ID Code: \_\_\_\_\_

Please write all solutions on this page (front and back if necessary).