

**Math 254-1 Exam 6: 10/27/8**

Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on **the attached page only** (front and back if necessary). Indicate clearly what work goes with which problem. Cross out work you do not wish graded; incorrect work can lower your grade. You may use this first page as scratch paper; keep it for your records. Show all necessary work in your solutions; if you are unsure, show it. Extra credit may be earned by handing in revised work in class on Wednesday 10/29; for details see the syllabus. Each problem is worth 10 points; your total will be doubled to fit the standard 100 point scale. You have approximately 30 minutes.

1. Carefully define the Linear Algebra term “independent”. Give two examples from  $\mathbb{R}^2$ .
2. In the vector space  $M_{2,3}$  of  $2 \times 3$  matrices, set:  
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \end{pmatrix}, B = \begin{pmatrix} 2 & 4 & 7 \\ 10 & 1 & 13 \end{pmatrix}, C = \begin{pmatrix} 1 & 2 & 5 \\ 8 & 2 & 11 \end{pmatrix}$$
Determine whether or not  $\{A, B, C\}$  is independent.
3. In the vector space  $\mathbb{R}_3[x]$  of polynomials of degree at most 3, set  $u_1 = x^3 + x^2 + 2x + 1$ ,  $u_2 = x^3 - x^2 + x + 1$ ,  $u_3 = x^3 + 5x^2 + 4x + 1$ ,  $u_4 = x^3 + 2x^2 + 3x + 4$ .  
Set  $S = \text{span}\{u_1, u_2, u_3, u_4\}$ . Find the dimension of  $S$ , and a basis.
4. In the vector space  $\mathbb{R}^2$ , set  $S = \{(1, 1), (4, 5)\}$ , a basis. Find the change-of-basis matrix from the standard basis to  $S$ , and use this matrix to find  $[(5, -3)]_S$ .
5. In the vector space  $\mathbb{R}^3$ , set  $T = \{(1, 1, 1), (0, 1, 2), (1, 1, 3)\}$ , a basis. Find  $[(1, 2, 2)]_T$ .

**Please hand in ONLY the second page; keep this first page.**

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Please write all solutions on this page (front and back if necessary).