

Math 254 Exam 6: 10/24/6

Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Extra credit may be earned by handing in revised work in class on Thursday 10/26; for details see the syllabus. Each problem is worth 10 points. You have approximately 30 minutes.

1. Carefully define the Linear Algebra term “independent”.
2. In the vector space $M_{2,3}$ of 2×3 matrices, set:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \end{bmatrix}, B = \begin{bmatrix} 2 & 4 & 7 \\ 10 & 1 & 13 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 & 5 \\ 8 & 2 & 11 \end{bmatrix}$$

Determine whether or not $\{A, B, C\}$ is independent.

3. In the vector space $P_3(x)$ of polynomials of degree at most 3, set $u_1 = x^3 + 3x^2 - 2x + 4$, $u_2 = 2x^3 + 7x^2 - 2x + 5$, $u_3 = x^3 + 5x^2 + 2x - 2$, $u_4 = 2x^3 + 6x^2 - 4x + 5$
Set $S = \text{span}\{u_1, u_2, u_3, u_4\}$. Find the dimension of S , and a basis.
4. In the vector space \mathbb{R}^2 , set $S = \{(1, 3), (1, 4)\}$, a basis. Find the change-of basis matrix from S to the standard basis, and use this matrix to find $[(5, -3)]_S$.
5. In the vector space \mathbb{R}^3 , set $T = \{(1, 1, 1), (0, 1, 2), (1, 1, 3)\}$, a basis. Find $[(1, 2, 2)]_T$.