Math 254 Exam 5: 10/17/6

Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Extra credit may be earned by handing in revised work in class on Thursday 10/19; for details see the syllabus. Each problem is worth 10 points. You have approximately 30 minutes.

1. Carefully define the Linear Algebra term "independent".

   The next three questions concern the matrix $A = \begin{bmatrix} 1 & 2 & 0 & \text{-1} \\ 2 & 6 & \text{-3} & \text{-3} \\ 3 & 10 & \text{-6} & \text{-5} \end{bmatrix}$.

2. Set $S = rowspace(A)$. Find a basis for S. Determine the dimension of S.

3. Set $T = columnspace(A)$. Find a basis for T. Determine the dimension of T.

4. Write the matrix equation $AX = 0$ as a homogeneous system of equations; find a basis for its solution space.

5. In the vector space $\mathbb{R}^3$, set $U = span\{(1,2,3),(5,-2,3)\}, V = span\{(-1,2,1)\}$. Determine the dimensions of each of $U, V, U \cap V, U + V$. Justify your answers.