Name:

Math 245 Spring 2013 Final

Please read the following directions.

PLEASE DO NOT BEGIN THE EXAM UNTIL INSTRUCTED TO DO SO.

Books and notes are permitted on this exam, but not calculators, computers, or other electronic aids. Please write legibly, with plenty of white space. Please print your name in large letters in the space provided. Please fit your answers in the designated areas. Show all necessary work in your solutions; if you are unsure, show it. Erase work you do not wish graded; incorrect work can lower your grade. There is no need to include scratch work, side calculations, dead ends, or "Givens/Goal" tables – only correct and complete answers.

Please choose ten of the following eleven problems to complete, and cross out the one you wish to skip. If you don't cross one out, I will choose one at random for you.

Problem	Your Grade	Max Grade
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
11		10
TOTAL		100

(1) [4.5.11] Let R_1 and R_2 be relations on A such that $R_1 \subseteq R_2$. Let S_1 be the symmetric closure of R_1 and S_2 be the symmetric closure of R_2. Prove that $S_1 \subseteq S_2$.

(2) [6.1.16] Prove that for all $n \in \mathbb{N}$, $2 \cdot 2^1 + 3 \cdot 2^2 + 4 \cdot 2^3 + \cdots + (n+1) \cdot 2^n = n2^{n+1}$.

(3) [similar to 3.6.8] Prove that for any $A \subseteq \mathbb{N}$, there is a unique $B \subseteq \mathbb{N}$ such that for every $C \subseteq \mathbb{N}$, $C \setminus B = C \cap A$.

(4) [similar to 4.2.12d] Let X, Y, Z be sets, R, S be relations from Y to Z and T be a relation from X to Y. Prove that $(R \cup S) \circ T = (R \circ T) \cup (S \circ T)$.

(5) [similar to 5.2.15] Suppose that S is an equivalence relation on A. Let $f : A \to A/S$ be defined by $f(x) = [x]_S$. Prove that f is onto.

(6) [similar to 6.2.11] Let $\mathscr{P}_3(A)$ denote the set of all subsets of A that have exactly 3 elements. Prove that if A has n elements then $\mathscr{P}_3(A)$ has $\frac{n(n-1)(n-2)}{6}$ elements.

(7) [similar to 4.1.12] Let A, B, C be sets. Suppose that $B \neq \emptyset$, and that $A \times B \subseteq B \times C$. Prove that $A \subseteq C$.

(8) [related to material in 5.1] Find a function $f : \mathbb{Z} \to \mathbb{Z}$ such that $f(x)$ is transitive. Then find a second, different, such function.

(9) [related to material in 6.5] Let R be the relation on $\mathbb{Z}$ given by

$$R = \{(x, x+2) : x \in \mathbb{Z}\} \cup \{(x, x-3) : x \in \mathbb{Z}\}$$

Prove that the transitive closure of R is $\mathbb{Z} \times \mathbb{Z}$.

(10) [related to material in 3.3] Let A be a set and let $\mathcal{F}, \mathcal{G}, \mathcal{H} \subseteq \mathscr{P}(A)$. Suppose that $\cup\mathcal{F} \in \mathcal{G}$, $\cup\mathcal{G} \in \mathcal{H}$, and $\cup\mathcal{H} \in \mathcal{F}$. Prove that $\cup\mathcal{F} = \cup\mathcal{G} = \cup\mathcal{H}$.

(11) [related to material in 4.3 and 4.4] Let R be the relation on $\mathbb{R}$ defined by xRy if $x^2 \leq y^2$. Determine whether or not R is reflexive, symmetric, antisymmetric, and/or transitive.

Course Survey

Please detach this page and hand in your comments anonymously.

Please comment on the initial problem-solving portions (roughly 20 mins) of each class:

Please comment on the oral presentation portions (roughly 55 mins) of each class:

Please comment on the structure before the midterm (problems assigned chronologically):

Please comment on the structure after the midterm (fixed groups, problems assigned “fairly”):

Please comment on the balance between the instructor speaking and you engaging with problems.

Any other comments or suggestions: