

Name:

RED ID:

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## Fall 2022 Math 245 Final Exam

Please read the following directions:

Please write legibly, with plenty of white space. Please **print** your name and REDID in the designated spaces above. Please fit your answers into the designated areas; material outside the designated areas (such as on this cover page) will not be graded. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. The first four questions are worth 8-16 points, and the remaining sixteen questions are worth 9-18 points. The maximum possible score is 352 out of 350, which is 100.6%; the minimum possible score is 176 out of 350, which is 50.3%. The use of notes, books, calculators, or other materials on this exam is strictly prohibited, except you may bring one 8.5"x11" page (both sides) with your handwritten notes. If you need scratch paper, you may use any blank space on your note sheet and on this front page. This exam will begin at 10:30 and will end at 12:30; pace yourself accordingly. Good luck!

Special exam instructions for HH-130:

1. Please stow all bags/backpacks/boards at the front of the room. All contraband, except phones, must be stowed in your bag. All phones and smartwatches must be silent, non-vibrating, and either in your pocket or stowed in your bag.
2. Please remain quiet to ensure a good test environment for others.
3. Please keep your exam on your desk; do not lift it up for a better look.
4. If you have a question or need the restroom, please come to the front. Bring your exam.
5. If you are done and want to submit your exam and leave, please wait until one of the designated exit times, listed below. Please do **NOT** leave at any other time. If you are sure you are done, just sit and wait until the next exit time, with this cover sheet visible.

Designated exam exit times:

10:50 "See you next semester"

11:10 "I wish I had studied more"

11:30 "One extra hour of drinking – worth it"

11:50 "Maybe this will be good enough"

12:10 "There is nothing more in my brain, let me out of here"

12:30 "I need every second I can get"

REMINDER: Use complete sentences.

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Problem 1. Carefully state the following definitions/theorems:

a. disjunction

b. conditional interpretation theorem

Problem 2. Carefully state the following definitions/theorems:

a. disjunctive syllogism theorem

b. converse

Problem 3. Carefully state the following definitions/theorems:

a. predicate

b. recurrence

Problem 4. Carefully state the following definitions/theorems:

a. equivalence class

b. chain

Problem 5. Let  $p, q, r$  be arbitrary propositions. Prove that  $p \rightarrow q, q \rightarrow r \vdash p \rightarrow r$ .

Problem 6. Prove or disprove the proposition:  $\forall x \in \mathbb{N}, \exists y \in \mathbb{Q}, x < |y - x| < 2x$ .

Problem 7. Let  $x \in \mathbb{R}$ . Prove that  $\lceil x \rceil$  is unique.

Problem 8. Prove the proposition:  $\forall n \in \mathbb{N}, 10^n > 3^n + 4^n$ .

Problem 9. Let  $a_n = 1 + \sin n$ . Prove or disprove that  $a_n = O(1)$ .

Problem 10. Find a partition of  $\mathbb{R}$  into three parts, where each part has infinite cardinality.

Problem 11. Suppose that  $R$  is a relation on  $S$  satisfying the property  $\forall x, y \in S, xRy \rightarrow yRx$ . Prove that  $\forall x, y \in S, xRy \leftrightarrow yRx$ .

Problem 12. Consider the relation  $R$  on  $\mathbb{Q}$  given by  $R = \{(x, y) : (x - y + 1)^2 = 1\}$ . Determine, with proof, whether or not  $R$  is an equivalence relation.

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Problem 13. Find, with justification, all solutions to the linear modular system given by  $\{x \equiv 2 \pmod{9}, x \equiv 0 \pmod{11}\}$ .

Problem 14. Consider these relations on  $\mathbb{N}$ :  $R_1 = \{(x, y) : x|(y-x)\}$  and  $R_2 = \{(x, y) : x|y\}$ . Prove or disprove that  $R_1 = R_2$ .

Problem 15. Consider the relation  $R_1 = \{(x, y) : x|(y-x)\}$  on  $\mathbb{N}$ . Prove or disprove that  $R_1$  is a partial order.

Problem 16. Let  $R$  be a partial order on some set  $S$ , and let  $T \subseteq S$ . Suppose that  $a, a'$  are both least in  $T$ . Prove that  $a = a'$ .

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Problem 17. Let  $R_1, R_2$  both be the usual order  $\leq$  on  $S = \{1, 2, 3\}$ . Let  $R$  be the product order on  $S \times S$ . Find, with justification, the width and height of  $R$ .

Problem 18. Find a relation on  $S = \{1, 2, 3\}$  that is left-definite but not right-definite.

Problem 19. Consider the function  $f : \mathbb{N}_0 \times \mathbb{N}_0 \rightarrow \mathbb{N}$  given by  $f((x, y)) = 2^x(2y + 1)$ . Prove or disprove that  $f$  is injective.

Problem 20. Suppose that  $f : \mathbb{Q} \rightarrow \mathbb{Q}$  is a function satisfying  $\forall x \in \mathbb{Q}, f(f(f(x))) = x$ . Prove that  $f$  is a bijection.