Name: RED ID:

Fall 2021 Math 245 Exam 2

Please read the following directions:

Please write legibly, with plenty of white space. Please **print** your name and REDID in the designated spaces above. Please fit your answers into the designated areas; material outside the designated areas (such as on this cover page) will not be graded. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. Each of the ten problems is worth 6-12 points. The use of notes, books, calculators, or other materials on this exam is strictly prohibited, except you may bring one 3"x5" card (both sides) with your handwritten notes. This exam will begin at 10:00 and will end at 10:50; pace yourself accordingly. Good luck!

Special exam instructions for HH-130:

1. Please stow all bags/backpacks/boards at the front or rear of the room. All contraband, except phones, must be stowed in your bag. All phones must be silent, non-vibrating, and either in your pocket or stowed in your bag.
2. Please remain quiet to ensure a good test environment for others.
3. Please keep your exam on your desk; do not lift it up for a better look.
4. If you have a question or need the restroom, please come to the front. Bring your exam.
5. If you are done and want to submit your exam and leave, please wait until one of the three designated exit times, listed below. Please do **NOT** leave at any other time. If you are sure you are done, just sit and wait until the next exit time, with this cover sheet visible.

Designated exam exit times:
10:20 "I need to work harder"
10:40 "I can't wait to get out of here"
10:50 "I need every second I can get"

REMINDER: Use complete sentences.

Problem 1. Carefully state the following definitions/theorems:

a. Proof by Reindexed Induction

b. Big Omega (Ω)

Problem 2. Carefully state the following definitions/theorems:

a. Nonconstructive Existence theorem

b. well-ordered by $<$

Problem 3. Prove or disprove: For all $x \in \mathbb{Z}$, there is at most one $y \in \mathbb{Z}$ with $x = 3y^2 + 1$.

Problem 4. Prove or disprove: For all $x \in \mathbb{R}$, $\lceil x \rceil \leq \lfloor x \rfloor + 1$.

Problem 5. Use mathematical induction to prove: For all $n \in \mathbb{N}$, $n! \geq n$.

Problem 6. Solve the recurrence with initial conditions $a_0 = 1, a_1 = 2$, and recurrence relation $a_n = -4a_{n-1} - 4a_{n-2}$ $(n \geq 2)$.

Problem 7. Let $b_n = 100 + 4n$. Prove or disprove that $b_n = O(n)$.

Problem 8. Prove that, for all $x \in \mathbb{R},\ |x| + |x-2| \geq 2$.

Problem 9. Prove: For all $n \in \mathbb{Z}$ with $n \geq 2$, that $F_n \geq 2F_{n-2}$.
(Here F_n denotes the Fibonacci numbers)

Problem 10. Use maximum element induction to prove that $\forall x \in \mathbb{N}, \exists n \in \mathbb{N}_0,\ 2^n \leq x < 2^{n+1}$.