Name:

Fall 2016 Math 245 Final Exam

Please read the following directions:
Please write legibly, with plenty of white space. Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. You may use a single page of notes, but no calculators or other aids. This exam will last 120 minutes; pace yourself accordingly. Please try to keep a quiet test environment for everyone. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
11.	5		10
12.	5		10
13.	5		10
14.	5		10
15.	5		10
16.	5		10
17.	5		10
18.	5		10
19.	5		10
20.	5		10
Total:	100		200

Problem 1. Carefully define the following terms:

a. factorial

b. floor of x

c. well-ordered by $<$

d. $a_n = O(b_n)$

e. $S = T$, for sets S, T

Problem 2. Carefully define the following terms:

a. Division Algorithm theorem

b. Double Negation theorem

c. Vacuous Proof theorem

d. Left-to-Right principle

e. Uniqueness Proof theorem

Problem 3. Carefully define the following terms:

a. De Morgan's Law (for sets) Theorem

b. $S \times T$ (for sets S, T)

c. set of arrival

d. transitive

e. equivalence relation

Problem 4. Carefully define the following terms:

a. equivalence class

b. partial order

c. interval poset

d. function

e. injection

Problem 5. Let $x \in \mathbb{R}$. Prove that if x is irrational, then $2 + \sqrt{x}$ is irrational.

Problem 6. Prove or disprove that $\mathbb{Q}\Delta\mathbb{Z} = \mathbb{Q}\Delta\mathbb{N}$.

Problem 7. Use Cantor's Theorem to prove that there is no largest set.

Problem 8. Find all integers x with $0 \leq x < 24$, that satisfy $4x \equiv 12 \pmod{24}$.

Problem 9. Let S be a set, R a reflexive relation on S, and $T \subseteq S$. Prove that $R|_T$ is reflexive.

Problem 10. Find an integer x with $0 \leq x < 5$, that satisfies $x \equiv 3^{1111} \pmod{5}$.

Problem 11. Let A, B, C be sets, and let f, g be surjective functions $f : A \to B$ and $g : B \to C$. Prove that $g \circ f$ is surjective.

Problem 12. Let p, q, r be propositions. Prove that $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.

Problem 13. Prove or disprove that $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, \exists z \in \mathbb{R}, (x < y) \rightarrow (x < z < y)$.

Problem 14. Consider the sequence given by $F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2} \quad (n \geq 2)$. Prove that $\forall n \in \mathbb{N}_0, F_n F_{n+1} = \sum_{i=0}^{n} F_i^2$.

For problems 15 and 16, consider $S = \{a, b, c, d, e\}$, together with the following relation: $R = \{(a, b), (b, e), (b, c), (d, b), (d, e), (a, a), (b, b), (c, c), (d, d), (e, e)\}$. You may assume that (R, S) is a poset; no need for you to prove this.

Problem 15. With (R, S) as above, find the width, height, all maximal elements, all greatest elements, all minimal elements, all least elements.

Problem 16. With (R, S) as above, find all linear extensions (you may just give their Hasse diagrams).

Problem 17. Let S be a set, and R a reflexive relation on S. Prove that $R \subseteq R \circ R$.

Problem 18. Let S, T be sets. Prove that $2^S \cap 2^T = 2^{S \cap T}$.

Problem 19. Consider $f : (1, +\infty) \to (1, +\infty)$ given by $f : x \mapsto \frac{x}{x-1}$. Prove that f is a bijection.

Problem 20. Solve the recurrence given by $a_0 = 1, a_1 = 5, a_n = a_{n-1} + 2a_{n-2} \quad (n \geq 2)$.