Name:

Fall 2015 Math 245 Final Exam

Please read the following directions:
Please write legibly, with plenty of white space. Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. You may NOT use your book, notes, calculators or other aids. This exam will last 120 minutes; pace yourself accordingly. Please leave **only** at one of the following designated times: 8:20am, 8:40am, 9:00am, 9:20am, 9:40am, 10:00am. At all other times, please stay in your seat, to ensure a quiet test environment. Good luck!

Problem	Min Score	Your Score	Max Score
1.	5		10
2.	5		10
3.	5		10
4.	5		10
5.	5		10
6.	5		10
7.	5		10
8.	5		10
9.	5		10
10.	5		10
11.	5		10
12.	5		10
13.	5		10
14.	5		10
15.	5		10
16.	5		10
17.	5		10
18.	5		10
19.	5		10
20.	5		10
Total:	100		200

Problem 1. Carefully define the following terms:

a. symmetric relation

b. transitive relation

c. reflexive relation

Problem 2. Carefully define the following terms:

a. function

b. inverse relation

c. poset

Problem 3. Carefully define the following terms:

a. power set

b. modus tollens

c. predicate

Problem 4. Carefully define the following terms:

a. factorial function, i.e. $n!$

b. event

c. conditional probability, i.e. $\Pr(A|B)$

Problem 5. Carefully define the following terms:

a. graph

b. digraph

c. loop

Problem 6. Carefully define the following terms:

a. subgraph

b. induced subgraph

c. spanning tree

Problem 7. Disprove the statement: $\forall x \in \mathbb{R}, \forall y \in \mathbb{Z}, (x < y) \to (\exists z \in \mathbb{Q}, x + z = y)$.

Problem 8. Let $A = \{n \in \mathbb{Z} : n|12\}, B = \{n \in \mathbb{Z} : n^2 < 16\}$. Compute $A \cap B$.

Problem 9. Calculate and simplify $\binom{1/4}{3}$.

Problem 10. We roll a fair die. Let A be the event of getting an even number, and B the event of getting one of $\{4, 5, 6\}$. Determine whether or not A, B are independent.

Problem 11. Compute the coefficient of x^5y^2 in $(x+3y)^7$.

Problem 12. Draw a graph whose adjacency matrix is $\left(\begin{smallmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{smallmatrix}\right)$.

Problem 13. Prove that every subgraph of a tree is a forest.

Problem 14. Determine for which $m, n \in \mathbb{N}$ the graph $K_{m,n}$ has an Eulerian path.

Problem 15. Prove by induction that $2^{3n} - 1$ is divisible by 7, for all $n \in \mathbb{N}$.

Problem 16. Let $n \in \mathbb{N}$. Suppose there is some $q \in \mathbb{Z}$ such that $n = 3q + 2$. Prove that n is not a perfect square.

Problem 17. Prove that K_3 is not a subgraph of $K_{4,5}$.

For problems 18,19 we ignore area codes; a "phone number" is seven digits.

Problem 18. "Phone numbers" consist of a three-digit exchange, and a four-digit subscriber number. The first digit of the exchange may not be 0 or 1, and the last two digits of the exchange may not both be 1. How many seven-digit "phone numbers" are possible?

Problem 19. California has 40 million people. Suppose each has their own "phone number". What does the extended/generalized pigeonhole principle tell us?

Problem 20. You play a game where you roll two fair dice, and win the larger of the two numbers. (e.g. if you roll 3 and 1 you win \$3). What is the probability you win \$5 or more?