Math 151 Fall 2011 Final Exam

Please read the following directions.

PLEASE DO NOT BEGIN THE EXAM UNTIL INSTRUCTED TO DO SO.

Books, notes, calculators, and other aids are not permitted on this exam. Please write legibly, with plenty of white space. Please put your answers in the designated areas. Show all necessary work in your solutions; if you are unsure, show it. Cross out work you do not wish graded; incorrect work can lower your grade. If you need additional paper, please use the back of the last page.

Problem	Your Grade	Max Grade
1		8
2		12
3		12
4		8
5		8
6		8
7		10
8		8
9		8
10		10
11		8
TOTAL		100

 $\overline{1. \text{ Evaluate } \int x \cos(4x) dx.}$

2. (a) Evaluate $\int \frac{\ln(x)}{x^2} dx$; (b) Determine whether the improper integral $\int_{e^2}^{\infty} \frac{\ln(x)}{x^2} dx$ converges or diverges, and its value in case of convergence.

3. (a) Determine the partial fraction decomposition of $\frac{2x^2+4}{x(x^2+4)}$; (b) Evaluate $\int \frac{2x^2+4}{x(x^2+4)} dx$.

4. Express $\int_{4}^{4\sqrt{3}} \sqrt{64 - x^2} dx$ as an integral in terms of powers of $\sin(u)$ and $\cos(u)$. You need not evaluate that integral.

5. Determine whether the improper integral $\int_3^6 \frac{1}{(x-3)^{3/2}} dx$ converges or diverges, and its value in case of convergence.

6. Use the method of disks to determine the volume of the solid that is obtained by revolving the region between the graph of $f(x) = \sqrt{\cos(x)}$ and the interval $[0, \pi/4]$ about the x-axis.

7. (a) Use an integrating factor to determine the general solution of $\frac{dy}{dt} = -ty(t) + 2t$; (b) Solve the initial value problem $\frac{dy}{dt} = -ty(t) + 2t$, y(0) = 6.

8. Derive the Taylor polynomial of order 2 for $f(x) = x^{3/4}$ based at 1 (i.e., in powers of x - 1).

9. Determine the radius and open interval of convergence of $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^n}{n^2} (x-4)^n$.

10. Determine whether the infinite series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{(n^4+1)^{1/4}}$ converges absolutely, converges conditionally, or diverges. HINT: The limit comparison test can be helpful in testing for absolute convergence.

11. (a) Plot the graph of $r = f(\theta) = 2\sin(\theta)$, where $0 \le \theta \le \pi$, in the θr -coordinate plane. Indicate the points at which f has maxima and minima and the points at which $f(\theta) = 0$; (b) Plot the graph of $r = 2\sin(\theta)$, where $0 \le \theta \le \pi$, in the xy-plane ($x = r\cos(\theta), y = r\sin(\theta)$). Indicate the points at which the graph intersects the x-axis and the y-axis.