A Taste of Modern Mathematics:

how to pay your rent and rig an election

Vadim Ponomarenko

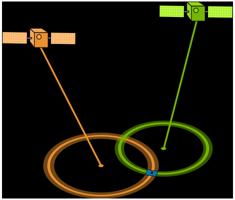
Department of Mathematics and Statistics
San Diego State University

MiraCosta College
April 25, 2014

http://www-rohan.sdsu.edu/~vadim/taste.pdf

San Diego State University

GPS: Global Positioning System



GPS Timeline

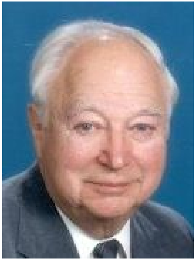
Today: Wide adoption

1973: Engineering breakthrough

Bradford Parkinson

(1935-)

Ivan Getting



(1912-2003)

Roger Easton

(1921-)

GPS Timeline

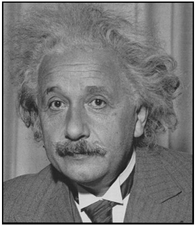
Today: Wide adoption

1973: Engineering breakthrough

Bradford Parkinson Ivan Getting Roger Easton

1905: Physics breakthrough: relativity

Albert Einstein



(1879-1955)

GPS Timeline

Today: Wide adoption

1973: Engineering breakthrough

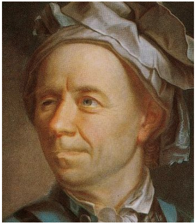
Bradford Parkinson Ivan Getting Roger Easton

1905: Physics breakthrough: relativity

Albert Einstein

1756: Mathematics breakthrough: wave equation

Leonhard Euler



(1707-1783)

SAN DIEGO STATE UNIVERSITY

GPS Timeline

Today: Wide adoption

1973: Engineering breakthrough

Bradford Parkinson Ivan Getting Roger Easton

1905: Physics breakthrough: relativity

Albert Einstein

1756: Mathematics breakthrough: wave equation

Leonhard Euler

250 years between theorem and your pocket

Moral of the GPS Story

Math research from past 200 years: mostly invisible

Is there a lot of “invisible” math research being done?

YES!!

last 50 years > 150 years before that > all previous

Moral of the GPS Story

Math research from past 200 years: mostly invisible

Is there a lot of “invisible” math research being done?

YES!!

last 50 years > 150 years before that > all previous

Moral of the GPS Story

Math research from past 200 years: mostly invisible

Is there a lot of “invisible” math research being done?

YES!!

last 50 years > 150 years before that > all previous

Moral of the GPS Story

Math research from past 200 years: mostly invisible

Is there a lot of “invisible” math research being done?

YES!!

last 50 years > 150 years before that > all previous

Today's Talk

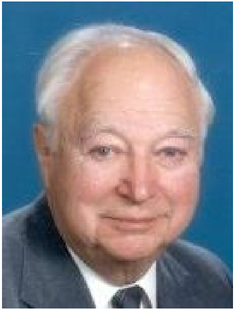
Two topics: Fair Division, Voting

Each reconciles individual ideas into a group decision.

The setup

Three friends:

Brad



Ivan

Roger

Rent = $1,500

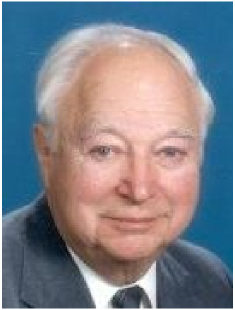
Three bedrooms: big, L-shaped, small

“Fair” means nobody thinks they’re getting a bad deal.

The setup

Three friends:

Brad



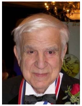
Ivan

Roger

Rent = $1,500
Three bedrooms: big, L-shaped, small
“Fair” means nobody thinks they’re getting a bad deal.

The preference matrix

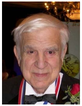
Step 1: Each friend decides what each room is worth *to him*.

			
	Brad	Ivan	Roger
Big room:	500	530	600
L-shaped room:	500	520	550
Small room:	500	450	350

Note: Each friend's total (columns) should be at least $1,500.
"Fair" means nobody is forced to pay more than he feels the room is worth.

The preference matrix

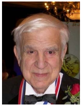
Step 1: Each friend decides what each room is worth *to him*.

	 Brad	 Ivan	 Roger
Big room:	500	530	600
L-shaped room:	500	520	550
Small room:	500	450	350

Note: Each friend's total (columns) should be at least $1,500.
"Fair" means nobody is forced to pay more than he feels the room is worth.

Assigning rooms

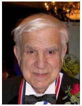
Step 2: Give out the rooms to maximize total money paid.

	 Brad	 Ivan	 Roger
Big room:	500	530	600
L-shaped room:	500	520	550
Small room:	500	450	350

Total money paid = \$500+\$520+\$600 = \$1,620.
Condition: Each person must have exactly one room.
Note: \$120 surplus!

Assigning rooms

Step 2: Give out the rooms to maximize total money paid.

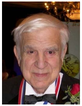
  

	Brad	Ivan	Roger
Big room:	500	530	600
L-shaped room:	500	520	550
Small room:	500	450	350

Total money paid = \$500+\$520+\$600 = \$1,620.
Condition: Each person must have exactly one room.
Note: \$120 surplus!

Returning the surplus

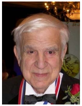
Step 3: Return the surplus.

	 Brad	 Ivan	 Roger
Big room:	500	530	600
L-shaped room:	500	520	550
Small room:	500	450	350

Simplest method (1948): divide surplus evenly, $40 to each.

Bronisław Knaster (1893-1980)

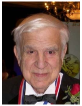
Is “fair” enough?

	 Brad	 Ivan	 Roger
Big room:	500	530	600
L-shaped room:	500	520	550
Small room:	500	450	350

Knaster’s method:
Brad pays $460, Ivan pays $480, Roger pays $560.

Although fair, this division leads to envy: Roger would like to switch with Ivan, since Roger feels that $480 is a better deal for the L-shaped room than he’s getting for the big room.

Is "fair" enough?

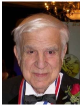
	 Brad	 Ivan	 Roger
Big room:	500	530	600
L-shaped room:	500	520	550
Small room:	500	450	350

Knaster's method:
Brad pays \$460, Ivan pays \$480, Roger pays \$560.

Although fair, this division leads to envy: Roger would like to switch with Ivan, since Roger feels that \$480 is a better deal for the L-shaped room than he's getting for the big room.

Envy-free division





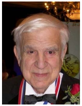
	Brad	Ivan	Roger
Big room:	500	530	600
L-shaped room:	500	520	550
Small room:	500	450	350

Can we distribute the surplus differently, to eliminate envy?

Brad pays $470, Ivan pays $490, Roger pays $540.
An envy-free division is always possible.

(Haake Raith Su 2002)  ← new prez of MAA

Envy-free division

	Brad	Ivan	Roger
Big room:	500	530	600
L-shaped room:	500	520	550
Small room:	500	450	350

Can we distribute the surplus differently, to eliminate envy?

Brad pays \$470, Ivan pays \$490, Roger pays \$540.
An envy-free division is always possible.

(Haake Raith Su 2002)  ← new prez of MAA

San Diego State University

Voting Systems

Each voter has preferences, giving an ordering of candidates.

A voting system combines these into a mutual ordering.

Puzzle: Suppose everyone prefers candidate A to B. You get to design the voting system, but you can't cheat (or even vote yourself). How can you make B win?

Voting Systems

Each voter has preferences, giving an ordering of candidates.

A voting system combines these into a mutual ordering.

Puzzle: Suppose everyone prefers candidate A to B. You get to design the voting system, but you can't cheat (or even vote yourself). How can you make B win?

Duggan-Schwartz Theorem

John Duggan: 

Thomas Schwartz: 

Thm (2000):

Every reasonable voting system will sometimes encourage dishonesty.

Duggan-Schwartz Theorem

John Duggan: 

Thomas Schwartz: 

Thm (2000):

Every reasonable voting system will sometimes encourage dishonesty.

Duggan-Schwartz Theorem, cont.

What's a 'reasonable' voting system?

1. At least 3 candidates.
2. More voters than winning candidates.
3. All voters count equally.
4. Every candidate can win (with enough votes).

Voting Systems

Plurality: Everybody votes for only one choice.

really bad

Approval: Everybody approves/disapproves of each candidate.

pretty good, simple

Several others...

Voting Systems

Plurality: Everybody votes for only one choice.

really bad

Approval: Everybody approves/disapproves of each candidate.

pretty good, simple

Several others...

Voting Systems

Plurality: Everybody votes for only one choice.

really bad

Approval: Everybody approves/disapproves of each candidate.

pretty good, simple

Several others...

Voting Systems

Plurality: Everybody votes for only one choice.

really bad

Approval: Everybody approves/disapproves of each candidate.

pretty good, simple

Several others...

Voting Systems

Plurality: Everybody votes for only one choice.

really bad

Approval: Everybody approves/disapproves of each candidate.

pretty good, simple

Several others...

Puzzle Solution

Puzzle: Suppose everyone prefers candidate A to B. You get to design the voting system, but you can't cheat (or even vote yourself). How can you make B win?

Introduce two new candidates that divide the electorate into thirds as follows:

One third of voters: $A >> B > X > Y$
One third of voters: $Y > A >> B > X$
One third of voters: $X > Y > A >> B$

Puzzle Solution

Puzzle: Suppose everyone prefers candidate A to B. You get to design the voting system, but you can't cheat (or even vote yourself). How can you make B win?

Introduce two new candidates that divide the electorate into thirds as follows:

One third of voters: $A >> B > X > Y$

One third of voters: $Y > A >> B > X$

One third of voters: $X > Y > A >> B$

Puzzle Solution, cont.

One third of voters: $A >> B > X > Y$

One third of voters: $Y > A >> B > X$

One third of voters: $X > Y > A >> B$

Voting system: We sequentially compare A, Y, X, B.

A vs. Y: Y wins by a landslide (2/3)

Y vs X: X wins by a landslide (2/3)

X vs B: B wins by a landslide (2/3)

Democracy triumphs! B wins the election.

Puzzle Solution, cont.

One third of voters: $A >> B > X > Y$
One third of voters: $Y > A >> B > X$
One third of voters: $X > Y > A >> B$

Voting system: We sequentially compare A, Y, X, B.

A vs. Y: Y wins by a landslide (2/3)
Y vs X: X wins by a landslide (2/3)
X vs B: B wins by a landslide (2/3)

Democracy triumphs! B wins the election.

Puzzle Solution, cont.

One third of voters: $A >> B > X > Y$
One third of voters: $Y > A >> B > X$
One third of voters: $X > Y > A >> B$

Voting system: We sequentially compare A, Y, X, B.

A vs. Y: Y wins by a landslide (2/3)
Y vs X: X wins by a landslide (2/3)
X vs B: B wins by a landslide (2/3)

Democracy triumphs! B wins the election.

Puzzle Solution, cont.

One third of voters: $A >> B > X > Y$
One third of voters: $Y > A >> B > X$
One third of voters: $X > Y > A >> B$

Voting system: We sequentially compare A, Y, X, B.

A vs. Y: Y wins by a landslide (2/3)
Y vs X: X wins by a landslide (2/3)
X vs B: B wins by a landslide (2/3)

Democracy triumphs! B wins the election.

End

Thank you for enduring all the way through.