Fair Division

how to have happy roommates

Vadim Ponomarenko

Department of Mathematics and Statistics San Diego State University

> Science Sampler November 18, 2007

http://www-rohan.sdsu.edu/~vadim/fair-div.pdf





But first, a word from our sponsors...

Clive Dorman

SDSU, College of Sciences, Dept. of Math/Stat

Summer Research program REUT

You!





Outline

Toys

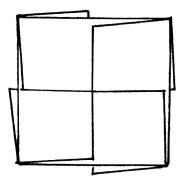
Fair Division

Bondage and Striptease





Hexaflexagons







The setup

Three friends:









Corbin

Jesse

Ryan





The setup

Three friends:







Corbin

Jesse

Ryan

Rent = \$1,500

Three bedrooms: big, L-shaped, small

"Fair" means nobody thinks they're getting a bad deal.





The preference matrix

Each friend privately decides what each room is worth to him.

			Los
	Corbin	Jesse	Ryan
Big room:	500	530	600
L-shaped room:	500	520	550
Small room:	500	450	350

Note: Each friend's total (columns) should be \$1,500. "Fair" means nobody is forced to pay more than he feels the room is worth.





The preference matrix

Each friend privately decides what each room is worth to him.

			Les SI
	Corbin	Jesse	Ryan
Big room:	500	530	600
L-shaped room:	500	520	550
Small room:	500	450	350

Note: Each friend's total (columns) should be \$1,500. "Fair" means nobody is forced to pay more than he feels the room is worth.





Assigning rooms

Give out the rooms to maximize total money paid.

			Les SI
	Corbin	Jesse	Ryan
Big room:	500	530	600
L-shaped room:	500	520	550
Small room:	(500)	450	350

Total money paid = \$500+\$520+\$600 = \$1,620.

Condition: Each person must have exactly one room.





Assigning rooms

Give out the rooms to maximize total money paid.

			Les SI
	Corbin	Jesse	Ryan
Big room:	500	530	600
L-shaped room:	500	520	550
Small room:	(500)	450	350

Total money paid = \$500+\$520+\$600 = \$1,620. Condition: Each person must have exactly one room.





Returning the surplus

			Les SI
	Corbin	Jesse	Ryan
Big room:	500	530	600
L-shaped room:	500	520	550
Small room:	500	450	350

Total money paid = \$500+\$520+\$600 = \$1,620, which gives \$120 surplus.

Simplest method (Knaster 1948): divide surplus evenly. Corbin pays \$460, Jesse pays \$480, Ryan pays \$560.





Returning the surplus

			L
	Corbin	Jesse	Ryan
Big room:	500	530	600
L-shaped room:	500	520	550
Small room:	500	450	350

Total money paid = \$500+\$520+\$600 = \$1,620, which gives \$120 surplus.

Simplest method (Knaster 1948): divide surplus evenly. Corbin pays \$460, Jesse pays \$480, Ryan pays \$560.





Is "fair" enough?

			Les SI
	Corbin	Jesse	Ryan
Big room:	500	530	600
L-shaped room:	500	520	550
Small room:	(500)	450	350

Knaster's method:

Corbin pays \$460, Jesse pays \$480, Ryan pays \$560.

Although fair, this division leads to envy: Ryan would like to switch with Jesse, since Ryan feels that \$480 is a better deal for the L-shaped room than he's getting for the big room.





Is "fair" enough?

			Los
	Corbin	Jesse	Ryan
Big room:	500	530	600
L-shaped room:	500	520	550
Small room:	500	450	350

Knaster's method:

Corbin pays \$460, Jesse pays \$480, Ryan pays \$560.

Although fair, this division leads to envy: Ryan would like to switch with Jesse, since Ryan feels that \$480 is a better deal for the L-shaped room than he's getting for the big room.





Envy-free division

			Los
	Corbin	Jesse	Ryan
Big room:	500	530	600
L-shaped room:	500	520	550
Small room:	500	450	350

Can we distribute the surplus differently, to eliminate envy?

Corbin pays \$470, Jesse pays \$490, Ryan pays \$540. An envy-free division is always possible. (Haake Raith Su 2002)





Envy-free division

			Los
	Corbin	Jesse	Ryan
Big room:	500	530	600
L-shaped room:	500	(520)	550
Small room:	500	450	350

Can we distribute the surplus differently, to eliminate envy?

Corbin pays \$470, Jesse pays \$490, Ryan pays \$540. An envy-free division is always possible. (Haake Raith Su 2002)





Final Thoughts on Fair Division

Same technique works for chores, inheritances.

It pays to be honest.

Each friend must be able to pay the difference.





Final Thoughts on Fair Division

Same technique works for chores, inheritances.

It pays to be honest.

Each friend must be able to pay the difference





Final Thoughts on Fair Division

Same technique works for chores, inheritances.

It pays to be honest.

Each friend must be able to pay the difference.





What to do with a dirty jacket

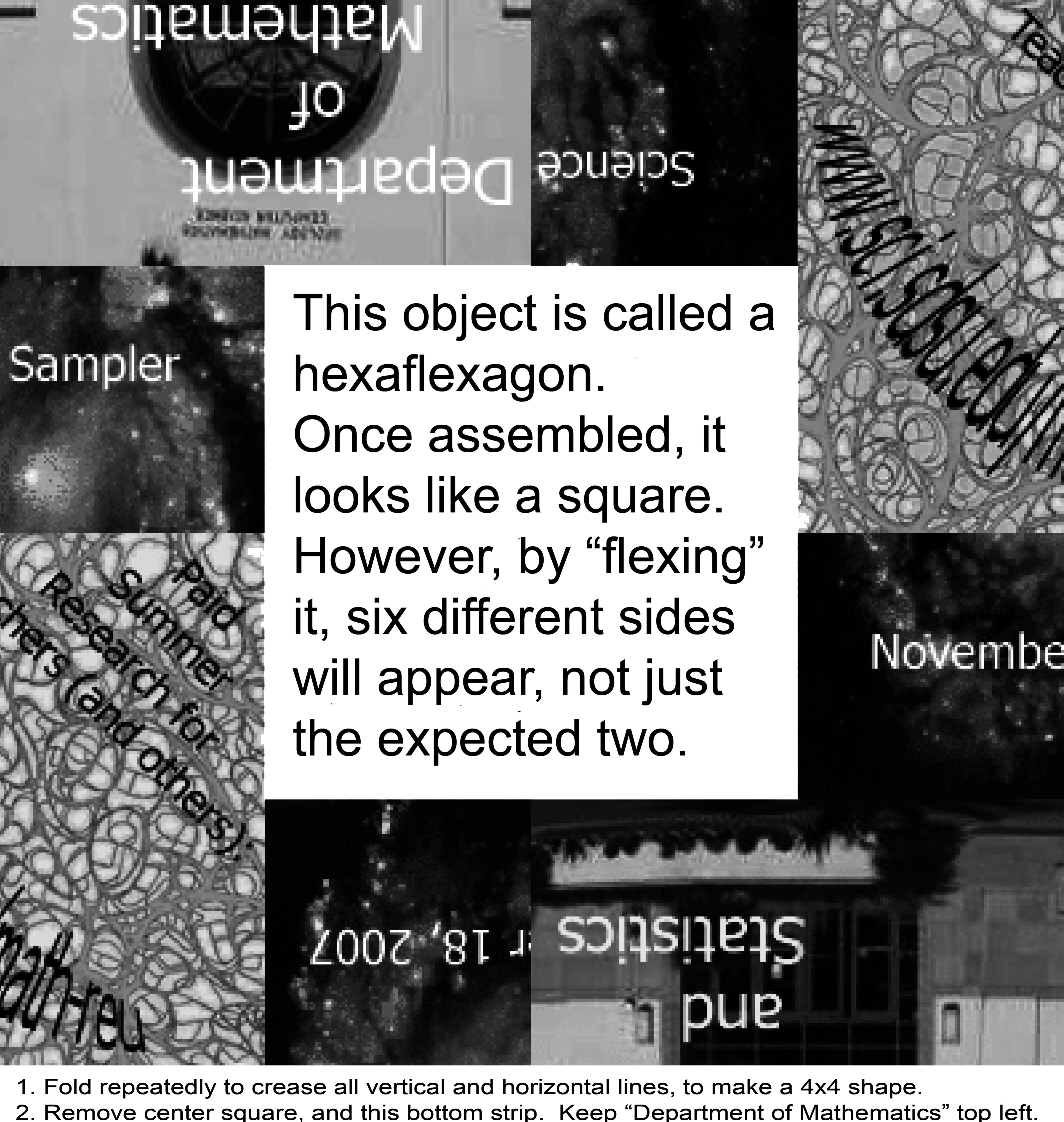
Clumsy me, I spilled mustard on my jacket while my hands were tied together with rope.

Questions:

- 1. Can I give my jacket to the dry cleaner?
- 2. Can I reverse my jacket?







- 2. Remove center square, and this bottom strip. Keep "Department of Mathematics" top left.
- 3. Fold leftmost column over (valley fold).



