MATH 623 Exam 1

Please read the following instructions. For the following exam you are free to use any papers or books you like, but no calculators or computers. Please turn in **exactly five** problems. You must do problems 1,2,3, and two more chosen from the remainder. Please write your answers on separate paper, make clear what work goes with which problem, and put your name or initials on every page. You have 75 minutes. Each problem will be graded on a 5-10 scale (as your quizzes), for a total score between 25 and 50. This will then be doubled for your exam score.

Turn in problems 1, 2, 3:

1. Suppose that $A \in M_n(\mathbb{F})$ has RREF of I_n. Prove that A may be written as the product of elementary matrices.
2. Let $J \in M_n(\mathbb{R})$ be the matrix all of whose entries are 1. Find $\sigma(J)$, and for each eigenvalue find a basis for the corresponding eigenspace.
3. Give an example of a matrix $M \in M_3(\mathbb{C})$ that is diagonalizable but not diagonal, and has fewer than 3 distinct eigenvalues.

Turn in exactly two more problems of your choice:

4. Calculate the adjugate and eigenvalues of $B = \left[\begin{smallmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{smallmatrix}\right]$.
5. Set $P_3(t)$ to be the set of all polynomials of degree at most 3, in variable t, with real coefficients. Find the rank and nullity of linear transformation $T : P_3(t) \to P_3(t)$ given by $T(f(t)) = t\frac{df(t)}{dt}$.
6. A matrix $A \in M_3(\mathbb{C})$ is a *square root* of B if $A^2 = B$. Prove that every diagonalizable $B \in M_3(\mathbb{C})$ has a square root.
7. Let $A \in M_3(\mathbb{C})$ be skew-symmetric. Prove that $P_A(t) = -P_A(-t)$, and that if λ is an eigenvalue of A, so is $-\lambda$.