

MATH601 Spring 2008
Exam 4: Cardinal Numbers

Please read all directions carefully. For this exam, you may not use any books, notes, or other aids apart from a calculator. Please write all solutions clearly and legibly, on separate paper, indicating what work applies to which problem. Cross out incorrect statements, this may improve your grade. Thoroughly justify your solutions. Your grade on each problem will be 25-50 points. If you wish, you may revise one of these problems for extra credit, due on Wednesday 3/26. Your grade on this problem will become the average of your original grade and the revised grade, rounded down. You have 35 minutes; good luck!

1. Let R, S, T be sets. Suppose that $|S| < |T|$ (this means that $|S| \leq |T|$ but $|S| \neq |T|$). Suppose further that $|R| = |S|$. Prove that $|R| < |T|$. You may use only the definitions of \leq and $=$, and the CSB theorem, but not any of the homework exercises.
2. Let S be “the largest possible set”, perhaps the set of all sets. Then surely $|S|$ is the largest possible cardinal. Find a larger cardinal, proving that there is no largest possible set.