MATH 579 Exam 2; 2/9/12

Please read the exam instructions.

No books or notes are permitted for this exam; calculators are permitted though. Please indicate what work goes with which problem, and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. You have 40 minutes. If you wish, when handing in your exam you may attach your extra credit problem. For more details, see the syllabus.

Choose three problems only from these five.

1. (5-8 points) Using mathematical induction, prove that for all positive integers n, we have $1 + 3 + 5 + \cdots + (2n - 1) = n^2$.
2. (5-10 points) Let $a_0 = a_1 = 1$, and $a_{n+2} = a_{n+1} + 5a_n$ for $n \geq 0$. Prove that $a_n \leq 3^n$ for all $n \geq 0$.
3. (5-10 points) Given $n \in \mathbb{N}$, the *alternating sum* of n is given as the units digit, minus the tens digit, plus the hundreds digit, minus the thousands digit, etc. For example, the alternating sum of $7,904,567$ is $7 - 6 + 5 - 4 + 0 - 9 + 7 = 0$. Prove that n is a multiple of 11 if and only if its alternating sum is a multiple of 11.
4. (5-10 points) Prove that for every triangulated simple polygon, it is possible to color each of its vertices red, blue, or green such that every triangle has its three vertices of different colors.
5. (5-12 points) The Fibonacci numbers are defined as $F_1 = F_2 = 1$, $F_{n+2} = F_{n+1} + F_n$ for $n \geq 1$. Let $\phi = \frac{1+\sqrt{5}}{2}$. Prove that $\frac{\phi^n}{3} < F_n < \phi^n$, for all $n \in \mathbb{N}$.