MATH 579 Final Exam

May 18, 2010

Please read the exam instructions.

Please write your answers on separate paper and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this sheet for your records. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. You are welcome to use your book, notes, and calculators; if you use an earlier result be sure to cite it. This exam is out of 60 points maximum. **Choose six of the following eight problems.**

1. (5-8 points) How many permutations $p \in S_3$ satisfy $p^2 = 1$?
2. (5-10 points) We select as many subsets of [100] as possible, with the restriction that any two subsets we choose must have at least one element in common. How many subsets can we get?
3. (5-10 points) Prove that for all $n \in \mathbb{N}$, $\sum_{k=0}^{n} \frac{1}{k+1}\binom{n}{k}(-1)^{k+1} = \frac{-1}{n+1}$.
   HINT: Carefully integrate the binomial theorem.
4. (5-10 points) Calculate $S(8, 4)$.
5. (5-10 points) How many ways are there to list $\{1, 1, 2, 2, 3, 4, 5\}$ so that no two consecutive positions have the same number?
6. (5-10 points) Prove that if your nemesis picks 1000 integers from [2000], some pair that he picked will have one a multiple of the other.
7. (5-12 points) Solve the recurrence given by $a_n = a_{n-1} + a_{n-2} - a_{n-3} + 1$ $(n \geq 3)$, $a_0 = a_1 = a_2 = 0$ using the methods of the supplementary section (characteristic polynomial etc.).
8. (5-12 points) Solve the recurrence given by $a_n = a_{n-1} + a_{n-2} - a_{n-3} + 1$ $(n \geq 3)$, $a_0 = a_1 = a_2 = 0$ using generating functions.