

MATH 579 Exam 2 Part I
Assigned 2/11/10, Due by classtime 2/16/10

Please read the exam instructions.

Please write your answers on separate paper and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this sheet for your records. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. You are welcome to use your book, notes, calculators, computers, etc. This problem is worth 10-20 points.

You may *NOT* discuss possible solutions to this exam with any human prior to submission. Violations of this policy will cause catastrophic course failure.

Part I: In the coin game, we place n coins in a row, each either heads or tails. We can remove any of the heads, leaving a gap, but when we do we flip any immediately adjacent coins. This means at most two coins get flipped, but often fewer because gaps destroy adjacency. We keep doing this until we run out of heads; we win if all the coins are gone. For example we could play $TTHH \rightarrow H \cdot TH \rightarrow H \cdot H \cdot \rightarrow H \cdot \dots \rightarrow \dots$ (we win), or we could play as $TTHH \rightarrow TT \cdot T$ (we lose). Prove that it is possible to win if and only if the number of initial heads is odd.