MATH 579 Final Exam: 5/19/9
Please read the exam instructions.

Please write your answers on separate paper, indicate clearly what work goes with which problem, and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. This exam is out of 60 points maximum.

PART I: Choose four problems only from the first six.

1. (5-8 points) A class is attended by n juniors, n seniors, and n graduate students. In how many ways can these students form n groups of three people if each group is to contain a junior, a senior, and a graduate student? [3.44]

2. (5-10 points) The sum of five positive real numbers is 100. Prove that therea re two numbers among them whose difference is at most 10. [1.17]

3. (5-10 points) Let $a_1 = 5$, and let $a_{n+1} = a_n^2$. Prove that the last n digits of a_n are the same as the last n digits of a_{n+1}. [2.28]

4. (5-10 points) We want to select as many subsets of $[n]$ as possible, such that no two selected subsets have one contained in the other. Prove that we can always select at least $2^n/n$ subsets. [4.48a]

5. (5-10 points) In how many ways can we list the digits $\{1, 1, 2, 2, 3, 4, 5\}$ so that two identical digits are never in consecutive positions? [7.21]

6. (5-12 points) A student has to take eight hours of classes a week, with fewer hours on Friday than on Thursday. How many ways can this be done? [5.19]

PART II: Choose two from these three problems.

7. (5-10 points) Solve the recurrence $a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3} + 6$, with $a_0 = 1, a_1 = 1, a_2 = 3$.

8. (5-10 points) You and I are doing n homework problems. I take the first portion, you take the remainder. I ask the prof for a hint on one of my problems, you ask for hints on two of yours. How many ways can all this be done?

9. Do both problems that you skipped from Part I. Your score will be the lower of the two. Be sure to indicate which two problems you are counting as problem 9.

Extra credit: Predict your grade on this exam. If close (within 2%), you'll earn a 1% bonus; if exactly right, you'll earn a 2% bonus.