MATH 579 Exam 2: 2/12/9

Please read the exam instructions.

Please write your answers on separate paper, indicate clearly what work goes with which problem, and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. You may earn extra credit by submitting by the next class period (Feb. 17), revised solutions to all six problems – for more details, please see the syllabus.

PART I: Choose three problems only from the first five.

1. (5-8 points) Let $a_0 = 3.1$, and let $a_{n+1} = \sqrt{a_n + 8}$ for $n > 0$. Prove that $3 < a_n < 3.5$ for all natural n.
2. (5-10 points) For all positive integers n, prove that $\sum_{i=1}^{n} i(i+1)(i+2) = \frac{n(n+1)(n+2)(n+3)}{4}$.
3. (5-10 points) Let $a_0 = 2, a_n = 2a_{n-1} + 3^n$ (for $n \geq 1$). Prove that $a_n = 3^{n+1} - 2^n$.
4. (5-10 points) Prove that a positive integer is divisible by 3 if and only if the sum of its digits is divisible by 3.
5. (5-12 points) We have a rectangular chocolate bar, shaped as an $m \times n$ chessboard, from chocolate squares. We wish to break the bar into its mn constituent squares, iteratively: break the rectangle, take one of the resulting rectangles and break it, etc. Prove that no matter how we proceed, it will take exactly $mn - 1$ breaks until we are done.

PART II:

6. (10-20 points) For all natural n, prove that $3^{(4\times 10^n)}$ ends in $\ldots\underbrace{00\cdots0}_{n}1$. For example, for $n = 2$, 3^{400} ends in $\ldots 001$.