

Math 579 Exam 2 (part I): 2/8/7

Please read the exam instructions.

Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Each problem is worth a minimum of 5 points, and a maximum that is indicated. You have 40 minutes. *Choose three problems.*

1. (8 points) Let $a_0 = 1$, and let $a_{n+1} = 3a_n + 6$ for all nonnegative integers n . Prove that $a_n = 4 \cdot 3^n - 3$.
2. (10 points) Prove that for all positive integers n , we have
$$1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2.$$
3. (10 points) Prove that a positive integer is divisible by 9 if and only if the sum of its digits is divisible by 9.

For the next two problems, recall that the Fibonacci numbers are defined as $F_1 = F_2 = 1$, $F_{n+2} = F_{n+1} + F_n$ for nonnegative integer n .

4. (10 points) Prove that $F_m \geq (1.3)^m$, for all integer $m \geq 4$.
5. (12 points) Prove that $F_1^2 + F_2^2 + \cdots + F_m^2 = F_m F_{m+1}$ for all natural m .