

MATH 579: Combinatorics
Homework 8: Due Nov.6

1. Find a closed form for the generating function for the sequence $1, -1, 1, -1, 1, -1, \dots$
2. Find a closed form for the generating function for the sequence $0, 0, 0, 1, -1, 1, -1, 1, -1, \dots$
3. Find a closed form for the generating function $\sum_{k \geq 0} (7k - 2)x^k$.
4. Find a closed form for the generating function for the sequence $0, 1, 4, 9, 16, 25, \dots$
5. Find a closed form for the generating function for the Fibonacci numbers.
Note: No need to find a closed form for the sequence.
6. Solve the recurrence given by $a_0 = 0, a_1 = 1, a_n = 4a_{n-2} \ (n \geq 2)$ using generating functions.
7. Solve the recurrence given by $a_0 = a_1 = 2, a_n = -2a_{n-1} - a_{n-2} \ (n \geq 2)$ using generating functions.
8. Solve the recurrence given by $a_0 = a_1 = 0, a_n = a_{n-1} + 2a_{n-2} + 3 \ (n \geq 2)$ using generating functions.
9. Count the number of solutions to $a + b + c = n$ in nonnegative integers a, b, c , such that a is a multiple of 3, $b \leq 2$, and $c \geq 1$. Find a closed form for the sequence, and compute explicitly the value for $n = 20$.
10. Count the number of solutions to $a + b + c = n$ in nonnegative integers a, b, c , such that a is even, $b \leq 4$, and $c \geq 1$. Find a closed form for the sequence, and compute explicitly the value for $n = 20$.
11. Find the generating function for how many ways there are of making n cents in change, out of pennies, nickels, dimes, and quarters. Then compute explicitly the value for $n = 111$. Note: No need to find a closed form for the sequence, use a computer to answer the specific question you have.
12. Consider the recurrence given by $c_0 = 1, c_{n+1} = \sum_{i=0}^n c_i \ (n \geq 0)$. Find a generating function and a closed form for the sequence. Hint: Consider $\frac{C(x)}{1-x}$.