

## MATH 579 Exam 1; 9/5/13

Please read the exam instructions.

No books or notes are permitted for this exam; calculators are permitted though. Please indicate what work goes with which problem, and put your name or initials on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Show all necessary work in your solutions; if you are unsure, show it. Simplify all numerical answers to be integers, if possible. You have 40 minutes. If you wish, when handing in your exam you may attach your extra credit problem. For more details, see the syllabus.

**Choose three problems only from these five.**

1. (5-8 points) 55 distinct integers are selected, all from  $[1, 100]$ . Prove that some pair differs by 12.
2. (5-10 points) 300 points are placed within a unit cube. Prove that you can choose some 12 of these, all within 0.6 of each other.
3. (5-10 points) Use the PHP to prove that there is some  $n \in \mathbb{N}$  such that  $44^n - 1$  is divisible by 13.
4. (5-10 points) 17 distinct integers are selected, all from  $[1, 33]$ . Prove that some pair among these has greatest common divisor 1.
5. (5-12 points) Prove that there is a positive integer  $n$  such that the distance from  $n\pi$  to the nearest integer is less than  $10^{-100}$ .