Math 524 Exam 7: 10/30/8

Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Each problem is worth 10 points. You have approximately 30 minutes.

1. Given any 2×2 matrix A, we consider the usual three systems, as below. If possible, produce five such matrices A, subject to the restrictions given.
   (I) $x(n) = Ax(n-1)$, (II) $dx/dt = Ax$, (III) $d^2x/dt^2 = Ax$.
   (a) (I) and (II) stable or neutral, (III) unstable
   (b) (I) and (III) stable or neutral, (II) unstable
   (c) (II) and (III) stable or netural, (I) unstable
   (d) all three systems unstable
   (e) all three systems stable

2. If possible, produce five Markov chains, subject to the following conditions:
   (a) irreducible, aperiodic, and recurrent
   (b) reducible, with at last one state periodic and at least one state transient
   (c) aperiodic and recurrent, but reducible
   (d) irreducible and recurrent, but at least one state periodic
   (e) irreducible and aperiodic, but at least one state transient

3. Consider the difference equation $x(n) = 5x(n-1) - 6x(n-2)$, with initial conditions $x(0) = 6, x(1) = 17$. Convert this into a 2×2 first-order problem, then solve it to get the general solution $x(n)$.

4. Consider the Markov chain pictured below. If the initial distribution is starting in A, i.e. $(1, 0, 0)^T$, find (approximately) the distribution after 12 time steps. You may use the approximation that $(0.9)^{12} \approx 2/7$.