

Math 524 Exam 1: 9/9/8

Please read the exam instructions.

Notes, books, papers, calculators and electronic aids are all forbidden for this exam. Please write your answers on **separate paper**, indicate clearly what work goes with which problem, and put your name on every sheet. Cross out work you do not wish graded; incorrect work can lower your grade, even compared with no work at all. Keep this list of problems for your records. Show all necessary work in your solutions; if you are unsure, show it. Each problem is worth 10 points. You have approximately 30 minutes.

1. State the eight axioms of a vector space.
2. Using only the eight vector space axioms, prove that an element of a vector space has at most one additive inverse.
3. In \mathbb{R}^2 , is it possible to have a set of two vectors that is:
 - (a) independent and spanning
 - (b) not independent and spanning
 - (c) independent and not spanning
 - (d) not independent and not spanning
4. Find the general solution to the coupled system of differential equations given by $\frac{d^2x}{dt^2} = \begin{pmatrix} -1 & 5 \\ 5 & -1 \end{pmatrix} x$. Hint: try $y_1 = x_1 + x_2, y_2 = x_1 - x_2$.

NOTE: You may leave the constants as constants, you need not find them in terms of $x(0)$.