MATH 521B: Abstract Algebra Exam 2

Please read the following instructions. For the following exam you are free to use a calculator and any papers you like, but no books or computers. Please turn in **exactly six** problems. You must do problems 1-4, and two more chosen from 5-8. Please write your answers on separate paper, make clear what work goes with which problem, and put your name or initials on every page. You have 75 minutes. Each problem will be graded on a 5-10 scale (as your quizzes), for a total score between 30 and 60. This will then be multiplied by $\frac{5}{3}$ for your exam score.

Turn in problems 1,2,3,4:

For these first four problems we fix $G \leq SL(2,\mathbb{R})$, defined as $G = \{\left(\begin{smallmatrix} a & b \\ 0 & d \end{smallmatrix}\right) : a, b, d \in \mathbb{R}, ad = 1\}$, and $N = \{\left(\begin{smallmatrix} 1 & b \\ 0 & 1 \end{smallmatrix}\right) : b \in \mathbb{R}\}$.

1. Prove that $N \trianglelefteq G$.
2. For each $x \in G$, determine explicitly its equivalence class $[x]$, modulo N.
3. Prove that $N \cong \mathbb{R}$.
4. Prove that $G/N \cong \mathbb{R}^\times$.

Turn in exactly two more problems of your choice:

5. Fix a group G, with $N \trianglelefteq G$. Suppose that $[G : N] = 20$. Prove that $a^{20} \in N$, for all $a \in G$.
6. Fix abelian group G, with $|G| = 2k$, and k odd. Prove that G has exactly one element g with $|g| = 2$.
7. Fix groups G, H, and suppose $A \trianglelefteq G$ and $B \trianglelefteq H$. Prove that $(A \times B) \trianglelefteq (G \times H)$.
8. Fix a group G. Set $A = \{K : |K| = 20,\ K \leq G\}$, the set of all subgroups of order 20, and assume that A is nonempty. Set $N = \bigcap_{K \in A} K$. Prove that $N \trianglelefteq G$. You need not prove that $N \leq G$.

You may also turn in the following (optional):

9. Describe your preferences for your next group assignment. (will be kept confidential)