MATH 521A: Abstract Algebra

Homework 10: Due Dec. 7

1. Prove that $(6, 15, 27) = (3)$ in $\mathbb{Z}$.
2. Find all ideals of $\mathbb{Z}_{12}$. Determine which of these are principal, maximal, and prime.
3. Suppose I, J are ideals of some ring R. Prove that $I \cap J$ and $I + J$ are both ideals of R.
4. Let R be a field. Prove that its only ideals are (0) and R.
5. Let R be a ring, and $a \in R$. Set $I = \{b \in R : ab = 0\}$. Prove that I is an ideal of R.
6. Calculate simple forms for the elements of the ideal $I = (6x, 10)$ in $R = \mathbb{Z}[x]$. Is it principal? Maximal? Prime?
7. Calculate simple forms for the elements of the ideal $I = (6x, 10x)$ in $R = \mathbb{Z}[x]$. Is it principal? Maximal? Prime?
8. Prove that $\mathbb{Z}/20\mathbb{Z} \cong \mathbb{Z}_{20}$. Some people prefer to write $\mathbb{Z}/20\mathbb{Z}$ instead of $\mathbb{Z}_{20}$.
9. Let I, K be ideals in R, with $K \subseteq I$. Prove that $I/K = \{x + K : x \in I\}$ is an ideal in $R/K = \{x + K : x \in R\}$.