Name:

Math 254 Spring 2014 Final Exam

Please read the following directions:
Please print your name in the space provided, using large letters, as "First LAST". Books, notes, calculators, and other aids are not permitted on this exam. Please write legibly, with plenty of white space. Please put your answers in the designated areas. Show all necessary work in your solutions; if you are unsure, show it. Cross out work you do not wish graded; incorrect work can lower your grade. All problems are worth 5-10 points; your total will be scaled to the standard 100 point scale. You have approximately 120 minutes. The back of the exam may be used for scratch paper, if necessary.

1. Carefully state the definition of "span", as it applies to vector spaces. Give two different sets that have the same span, drawn from $P_2(t)$.

2. Carefully state the definition of "subspace". Give two examples from within $M_{2,2}$.

3. Carefully state the definition of "basis". Give two examples from $\mathbb{R}^3$.

Problems 4 – 6 all concern matrix $A = \begin{pmatrix} 1 & 0 & 3 & 1 \\ 1 & 1 & 2 & 2 \\ 2 & 1 & 5 & 3 \\ 2 & 3 & 3 & 5 \end{pmatrix}$.

4. Put A in row canonical form.

5. Use your solution to (4) to find all solutions to $A \begin{pmatrix} a & b & c & d \end{pmatrix}^T = \begin{pmatrix} 0 & 0 & 0 & 0 \end{pmatrix}^T$.

6. Use your solution to (4) to find a basis for $Span(S)$, where
$S = \{(1, 0, 3, 1), (1, 1, 2, 2), (2, 1, 5, 3), (2, 3, 3, 5)\}$.

Problems 7-9 all concern the function $T : M_{2,2} \to M_{2,2}$ given by $T(\left[\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right]) = \left[\begin{smallmatrix} b & a \\ 0 & 2c \end{smallmatrix}\right]$.

7. Prove that T is a linear transformation.

8. Find the representation $[T]_E$, for the standard basis E of $M_{2,2}$.

9. Find the characteristic polynomial and eigenvalues of T.

Problems 10-12 all concern vector space $P_2(t)$ with inner product given by $\langle u, v\rangle = \int_0^1 u(t)v(t)dt$.

10. Find the angle between $u(t) = 1$ and $v(t) = t$.

11. Use Gram-Schmidt to find an orthogonal basis for $P_1(t)$, viewed as a subspace of $P_2(t)$.

12. Find the Fourier coefficients of $u(t) = 2 + 3t$ with respect to the basis you found in (11).

13. Suppose that $A \in M_{n,n}$. Prove that if $A = A^{-1}$, then $A^4 = A^{10}$.

14. Prove or find a counterexample for the following statement:
If $A, B, P \in M_{n,n}$, P is invertible, and $A = PB$, then A, B must have the same eigenvalues.

15. Let $A \in M_{m,n}$. Prove that the set of all solutions to $AX = 0$ is a subspace of $\mathbb{R}^n$.