Name:

Math 254 Fall 2014 Final Exam

Please read the following directions:
Please print your name in the space provided, using large letters, as "First LAST". Books, notes, calculators, and other aids are not permitted on this exam. Please write legibly, with plenty of white space. Please put your answers in the designated areas. Show all necessary work in your solutions; if you are unsure, show it. Cross out work you do not wish graded; incorrect work can lower your grade. All problems are worth 5-10 points; your total will be scaled to the standard 100 point scale. You have approximately 120 minutes. The back of the exam may be used for scratch paper, if necessary.

1. Carefully state the definition of "dependent". Give two different dependent sets, drawn from $P_2(t)$.

2. Carefully state the definition of "span". Give two different sets, each drawn from $P_2(t)$, with the same span.

3. Carefully state the definition of the standard vector space $\mathbb{R}^n$. Is it true that $\mathbb{R}^2 \subseteq \mathbb{R}^3$?

Problems 4 – 6 all concern matrix $A = \begin{pmatrix} 1 & 2 & 3 & 2 & 1 \\ 2 & 4 & 2 & 4 & 2 \\ 1 & 2 & 4 & 2 & 1 \end{pmatrix}$.

4. Put A in row canonical form. Be sure to justify each step.

5. Find bases for each of $rowspace(A), colspace(A), nullspace(A)$.

6. Set $X = \begin{pmatrix} a & b & c & d & e \end{pmatrix}^T$. Find all solutions to $AX = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$. Hint: $(-1, 0, 1, 0, 0)$ is one such.

Problems 7,8 both concern the basis $B = \{(1,2),(2,3)\}$ for $\mathbb{R}^2$.

7. Find the change-of-basis matrices Q_{EB} and Q_{BE}.

8. Use the Gram-Schmidt process on B to find an orthogonal basis for $\mathbb{R}^2$, using the non-standard inner product $\langle x, y\rangle = x^T \left(\begin{smallmatrix} 1 & 1 \\ 1 & 3 \end{smallmatrix}\right) y$.

9. Problems 9-12 all concern matrix $M = \left(\begin{smallmatrix} 2 & 2 \\ 2 & 5 \end{smallmatrix}\right)$. Find the eigenvalues and a basis for each eigenspace.

Problems 9-12 all concern matrix $M = \left(\begin{smallmatrix} 2 & 2 \\ 2 & 5 \end{smallmatrix}\right)$.

10. Is M diagonalizable?

Is M invertible?

Is M positive definite?

Is M positive semidefinite?

Is M singular?

11. Find matrices S, J such that $M = SJS^{-1}$, where S is invertible and J is in Jordan canonical form.

12. Find matrix R such that $M = R^T I R$, where R is invertible and $I = \left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right)$.

13. Prove that every invertible matrix equals the product of certain elementary matrices.

14. Let A be a square matrix, whose second row is five times its first row. Prove that $|A| = 0$.

15. Let U, V be vector spaces, and $F : U \to V$ a linear map. Suppose that F is a bijection; hence $F^{-1} : V \to U$ is well-defined and a bijection. Prove that F^{-1} is a linear map.