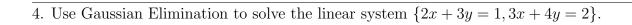
Math 254 Fall 2013 Final Exam

Please read the following directions:

Please print your name in the space provided, using large letters, as "First LAST". Books, notes, calculators, and other aids are not permitted on this exam, apart from a single $3"\times5"$ note card. Please write legibly, with plenty of white space. Please put your answers in the designated areas. Show all necessary work in your solutions; if you are unsure, show it. Cross out work you do not wish graded; incorrect work can lower your grade. All problems are worth 5-10 points; your total will be scaled to the standard 100 point scale. You have approximately 120 minutes. The back of the exam may be used for scratch paper, if necessary.

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1. Carefully state the definition of "dimension", as it applies to vector spaces. Gexamples, a four-dimensional one and an infinite-dimensional one.	live two
2. Carefully state the definition of "independent". Give two examples from $M_{2,2}$.	
3. Carefully state the definition of "subspace". Give two examples from within \mathbb{R}^3 .	



5. Use Cramer's Rule to solve the linear system
$$\{2x + 3y = 1, 3x + 4y = 2\}$$
.

6. In vector space
$$P_2(t)$$
, set $p_1(t) = 3 - t^2$, $p_2(t) = 3 + 2t + t^2$, $p_3(t) = t + t^2$. Find a basis for $Span(\{p_1, p_2, p_3\})$, and its dimension.

7. Prove that T is a linear transformation.

8. Find representation $[T]_E$, for standard basis $E = \{1, t, t^2\}$.

9. Find representation $[T]_S$, for basis $S = \{1 + t^2, t + t^2, t^2\}$.

$\overline{\ 10.}$ With T as in problems 7-9, find the characteristic polynomial of T and its eigenvalue $\overline{\ 10.}$	values.
Problems 11-12 both concern the vector space $M_{2,2}$, the standard inner product $X(A,B) = tr(B^TA)$, and vectors $X = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $Y = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$. 11. Find the angle between X and Y .	iven by
12. Find an orthogonal basis for $Span(\{X,Y\})$.	

