

Name: _____

Math 254 Fall 2011 Exam 6

Please read the following directions:

Books, notes, calculators, and other aids are not permitted on this exam. Please write legibly, with plenty of white space. Please put your answers in the designated areas. Show all necessary work in your solutions; if you are unsure, show it. Cross out work you do not wish graded; incorrect work can lower your grade. All problems are worth 5-10 points; your total will be scaled to the standard 100 point scale. You have approximately 30 minutes.

Extra credit may be earned by handing in revised work in class on Friday 10/28; for details see the syllabus. You will find this exam on the instructor's webpage later today.

1. Carefully state the definition of "basis". Give two examples, each from \mathbb{R}^1 .

2. Consider the vector space $\mathbb{R}[x, y]$ of polynomials in x, y . Let V be the subspace generated by basis $\{1, x, y, x^2, xy, y^2\}$. Set $S = \{(x+1)^2, (y-1)^2, (x+1)(y-1), (x+y)^2\}$. Determine if S is linearly independent.

3. In the vector space \mathbb{R}^2 , set $S = \{(2, 3), (5, 7)\}$, a basis. Find the change-of-basis matrix from the standard basis to S , and use this matrix to find $[(7, 10)]_S$.

4. In the vector space \mathbb{R}^3 , set $S = \{(2, 3, 1), (0, 1, 2), (3, 4, 0)\}$, a basis. Find the change-of-basis matrix from the standard basis to S , and use this matrix to find $[(7, 11, 3)]_S$.

5. In the vector space $M_{2,2}(\mathbb{R})$, the set of 2×2 matrices, set $A = \begin{pmatrix} 2 & 0 \\ -3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$, $C = \begin{pmatrix} 4 & -2 \\ -9 & 5 \end{pmatrix}$. Set $S = \text{Span}\{A, B, C\}$. Find the dimension of S , and a basis.